

BIG IDEAS

fractals

pythagoras

fibonacci

4colour

- * Fantastic Fractals
- * Pythagoras & his Amazing Triangle
- * Fibonacci the Blockhead
- * 4 Colour Theory

At the age of 4, Paul Erdos remarked to his mother,
'If you subtract 250 from 100, you get 150 below zero.'
Erdos could already multiply 3 and 4 digit numbers together in his head,
but no one had taught him about negative numbers.

'It was an independent discovery,' he recalls happily.
(Tierney, "Paul Erdos is in town. His brain is open", Science, Oct, 1984)

**Every child deserves the opportunity to make independent discoveries
for themselves.**

With these big ideas you can start in kindergarten with the visual concept and
then progress right through to year six
with more complex ideas including formulas.

Fantastic Fractals

Benoit B. Mandelbrot (20 November 1924 – 14 October 2010) was a mathematician, noted for developing a “theory of roughness” and “self-similarity” in nature and the field of fractal geometry he coined the word “fractal”.

Because of his access to IBM's computers, Mandelbrot was one of the first to use computer graphics to create and display fractal geometric images, leading to his discovering the Mandelbrot set in 1979, named in his honour. By doing so, he was able to show how visual complexity can be created from simple rules. He said that things typically considered to be “rough”, a “mess” or “chaotic”, like clouds or shorelines, actually had a “degree of order”

A fractal is a never-ending pattern that repeats itself at different scales. This property is called ‘Self-Similarity.’ Although fractals are very complex they are made by repeating a simple process.

There are many examples in nature, like trees, leaves, lightning, ferns, and broccoli. When you zoom in on these objects you see the same patterns over and over again.

In nature, you can eventually zoom in so far that the pattern doesn't exist anymore. But in mathematical fractals, no matter how far you zoom in, you will always see the same shape!

Google

Check out amazing fractal images - just Google them up.

View

2010: A Mandelbrot Odyssey (FractalNet HD)

<https://vimeo.com/9505449>

Great Fractal Videos From the Fractal Foundation

<http://fractalfoundation.org/videos/>

Things in nature with fractal patterns - what can you find. Fractal patterns are extremely familiar, since nature is full of fractals. For instance: trees, rivers, coastlines, mountains, clouds, seashells, hurricanes, etc. Abstract fractals – such as the Mandelbrot Set – can be generated by a computer calculating a simple equation over and over.

Fractals in nature - Grade 4 project

Play with these online Fractals

<http://fractalfoundation.org/resources/what-are-fractals/>

Complex Fractal Animations

<https://www.youtube.com/watch?v=8Z1EnoXh01Y>

Google Earth

The magnificent free program that allows you to explore anywhere on Earth. Zoom in to discover natural fractals such as mountains, rivers and coastlines. (Custom KMZ layers showing a special collection of natural fractals on the Earth.) Check out the collection at

Written by **Paul Bourke**

<http://paulbourke.net/fractals/googleearth/>

Sierpinski Triangle

The Sierpinski triangle is a fractal described in 1915 by Waclaw Sierpinski. It is a self similar structure that occurs at different levels of iterations, or magnifications.

These triangles below are similar because the triangles are all the same, just shrunk down to half the size. Notice that the angles on all the triangles are the same. Zooming in on them is very cool.

Watch the amazing video then make one yourself.

<https://www.youtube.com/watch?v=QsMvou5WlQ>

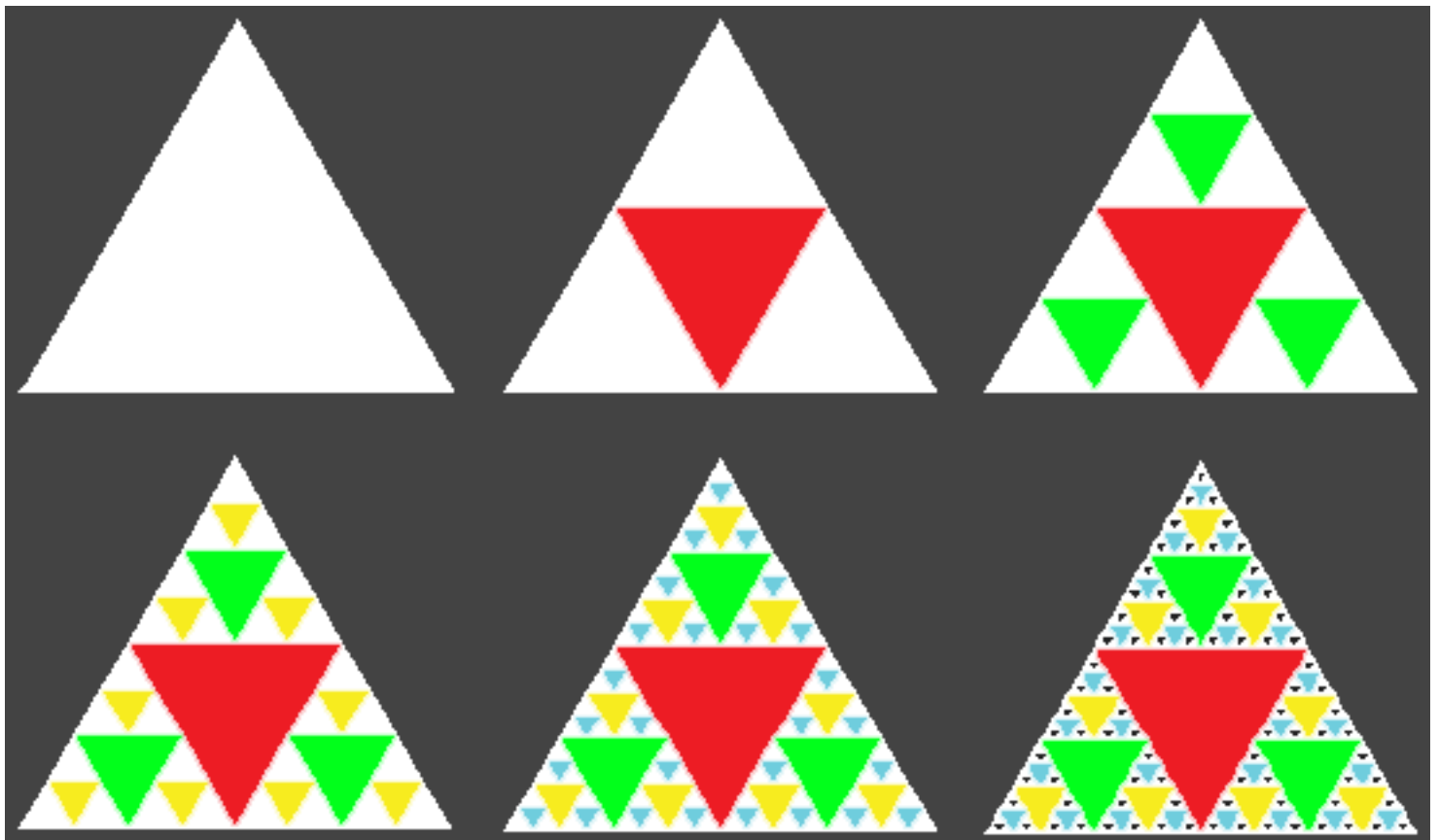
Let's participate in the Fractal Trianglethon!

<http://fractalfoundation.org/category/kids/>

How to draw a Sierpinski triangle - interactive

<http://www.shodor.org/interactivate/activities/SierpinskiTriangle/>

Use the outline on the following page to draw your own Sierpinski Triangle.

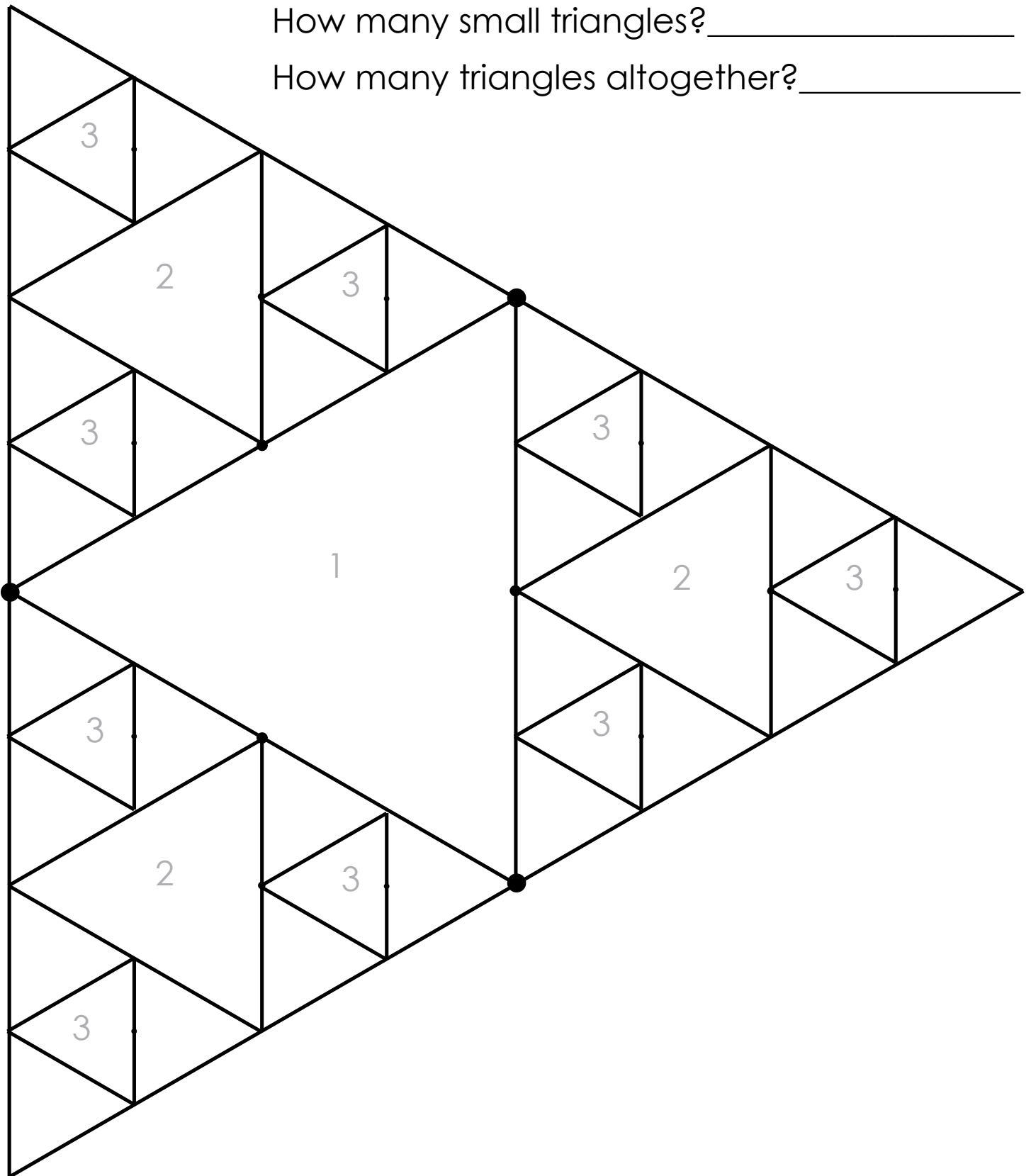


How many big triangles? _____

How many medium sized triangles? _____

How many small triangles? _____

How many triangles altogether? _____



Sierpinski Triangle

The 3 dot on the sides are the "midpoints" of the sides.

Connect these dots

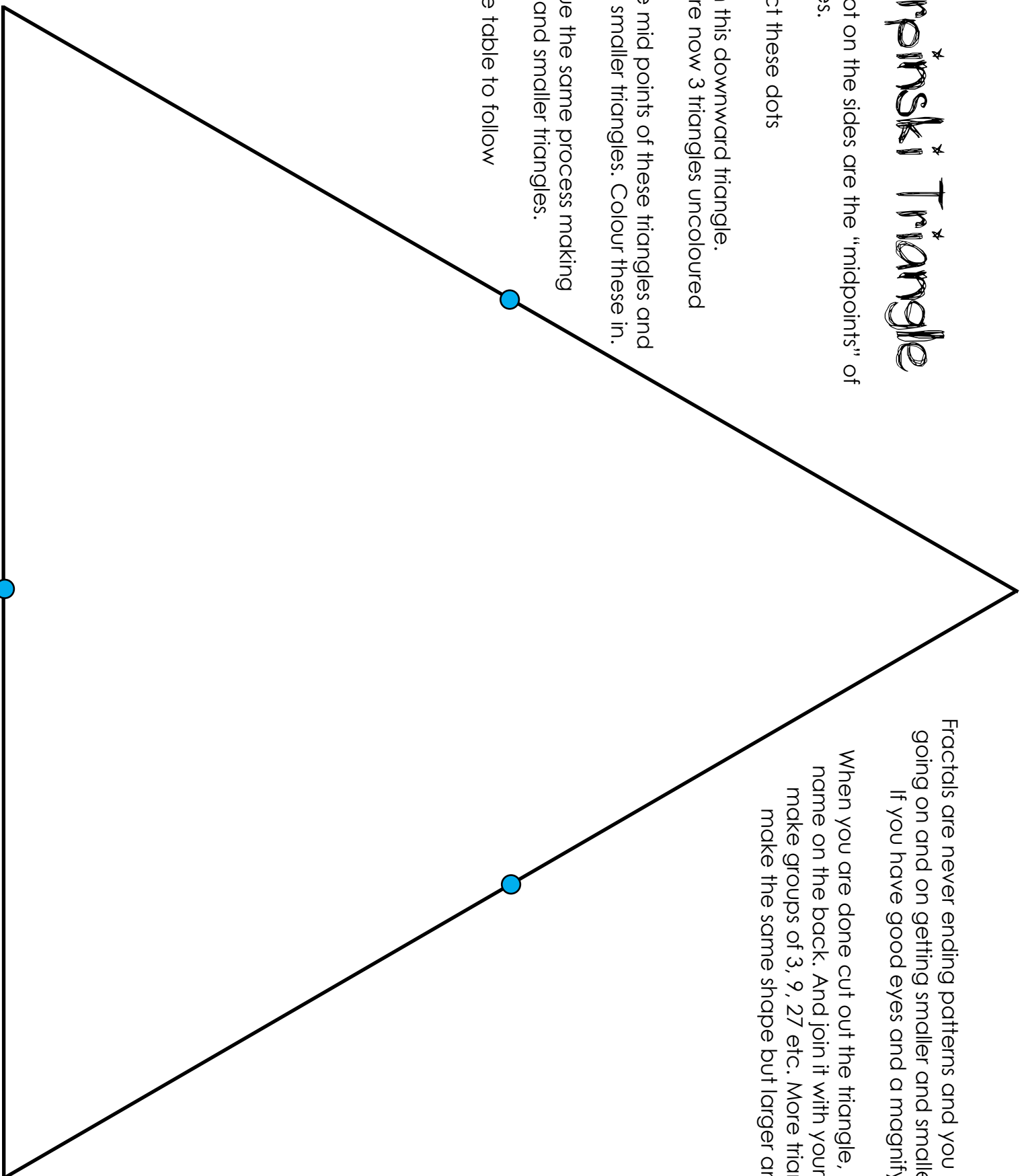
Color in this downward triangle.

There are now 3 triangles uncoloured

Find the mid points of these triangles and make 3 smaller triangles. Colour these in.

Continue the same process making smaller and smaller triangles.

Fill in the table to follow



Fractals are never ending patterns and you can keep going on and on getting smaller and smaller forever, If you have good eyes and a magnifying glass.

When you are done cut out the triangle, write your name on the back. And join it with your friends to make groups of 3, 9, 27 etc. More triangles that make the same shape but larger and larger.

Once you've made your triangle fill in the following table. Can you see the pattern? Can you work out the iterations(repetition of a process) up to 10

Iteration	Number of Triangles
0	1
1	3
2	9
3	
4	
5	
6	
7	
8	
9	
10	

Can you see the pattern? Can you write a formula for this equation?

Challenge

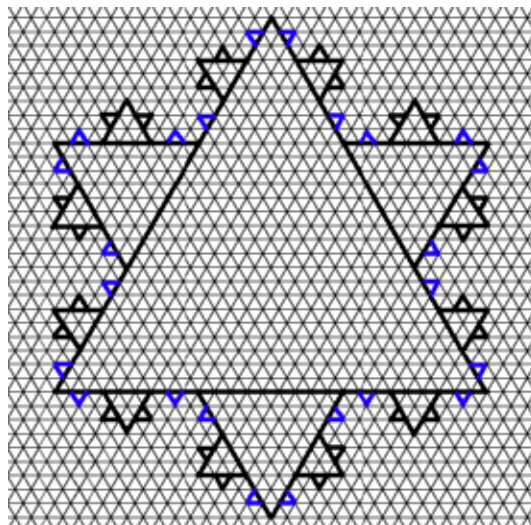
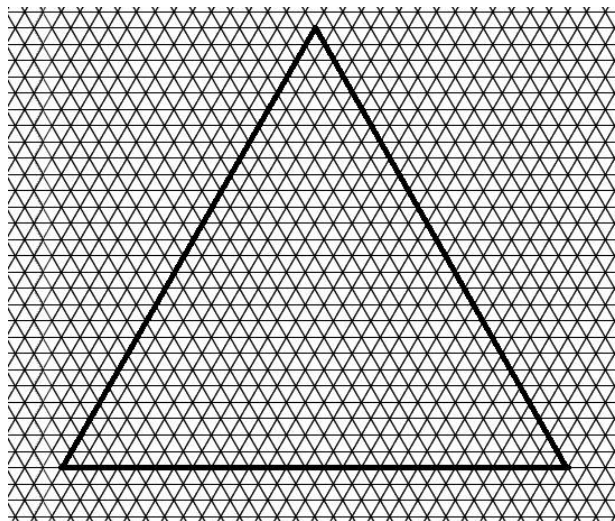
1. Check out iPad Fractal apps like Fast Fractal is FREE

<https://apps.apple.com/us/app/fast-fractal/id398923328>

2. Use Geometer's Sketchpad to construct a Sierpinski Triangle

Challenge

Let's investigate a specific fractal, the snowflake fractal, or Koch fractal, named after a Swedish mathematician, Helge von Koch. And work out the area and perimeter. To make this fractal we start with an equilateral triangle on special triangular graph paper. (You might have some of this lurking in a cupboard, if not print out the page to follow.)



Then goto

<https://www.youtube.com/watch?v=j6WYSdHuWgY>

to figure out the rest.

Goodluck and see how far you can go. Once you've worked out how to do it, see if you can investigate other fractals.

Check Out

Nico's Fractal Machine

<https://sciencevmagic.net/fractal/#0020,0055,6,1,1,0,1>

Fractal + App

Download Fractal + app from iTunes - FREE

<https://apps.apple.com/us/app/fast-fractal/id398923328>

Check out the App.

Select the Mandelbrot option

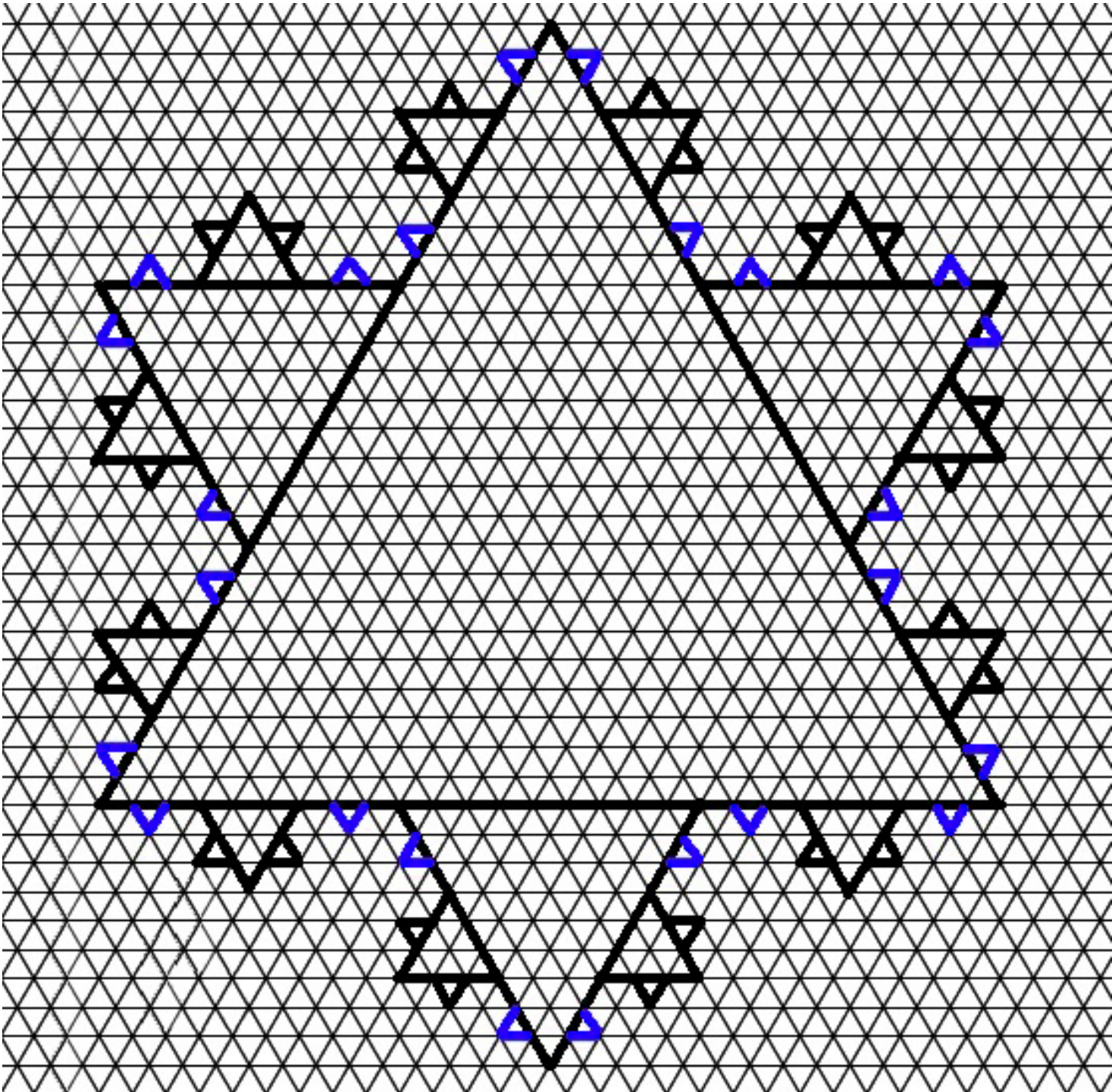
Change colours and iterations in the advanced settings. Change magnification.

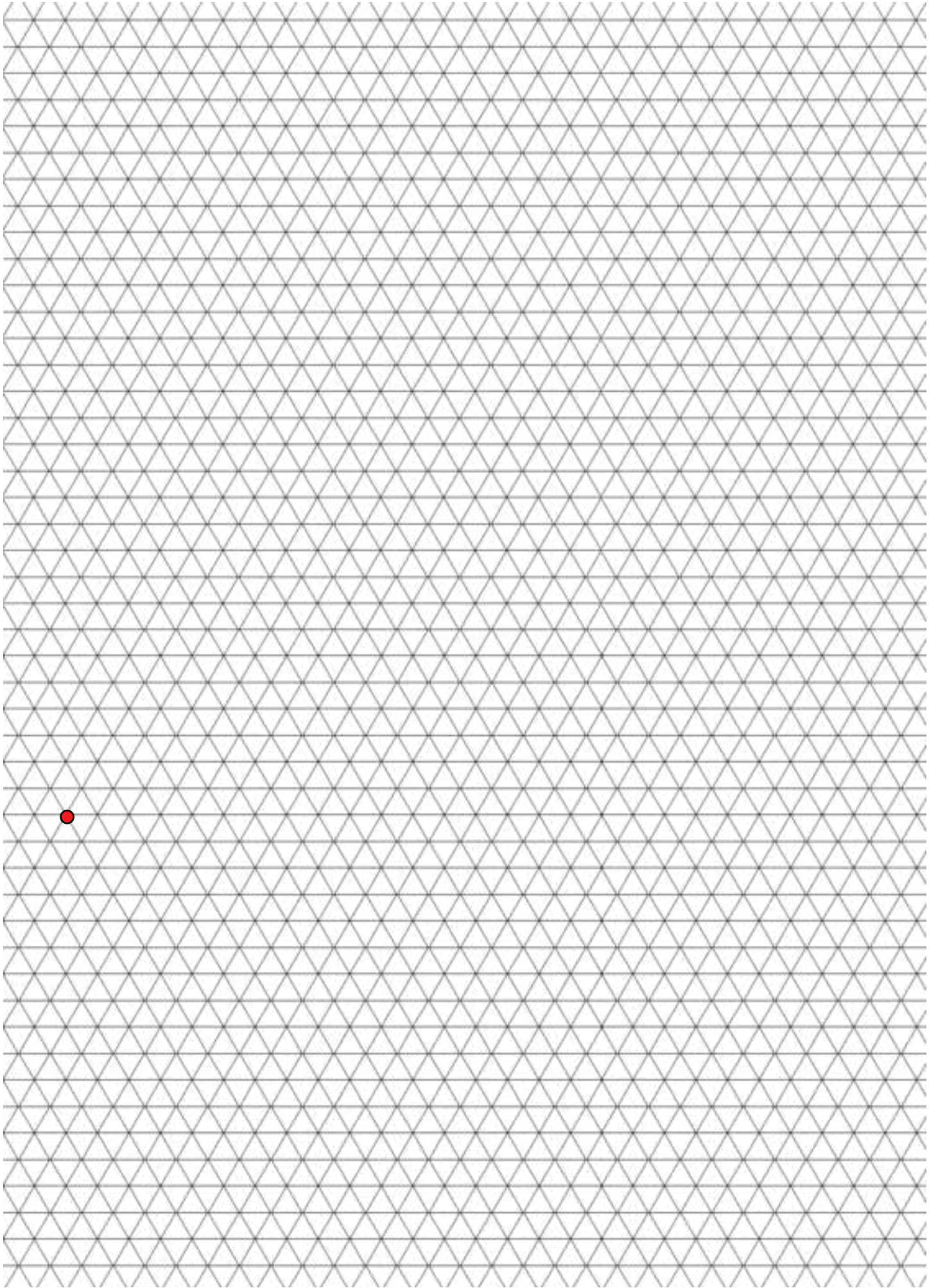
Choose your 8 favourite (2 each in group of 4) Magnifications and colours.

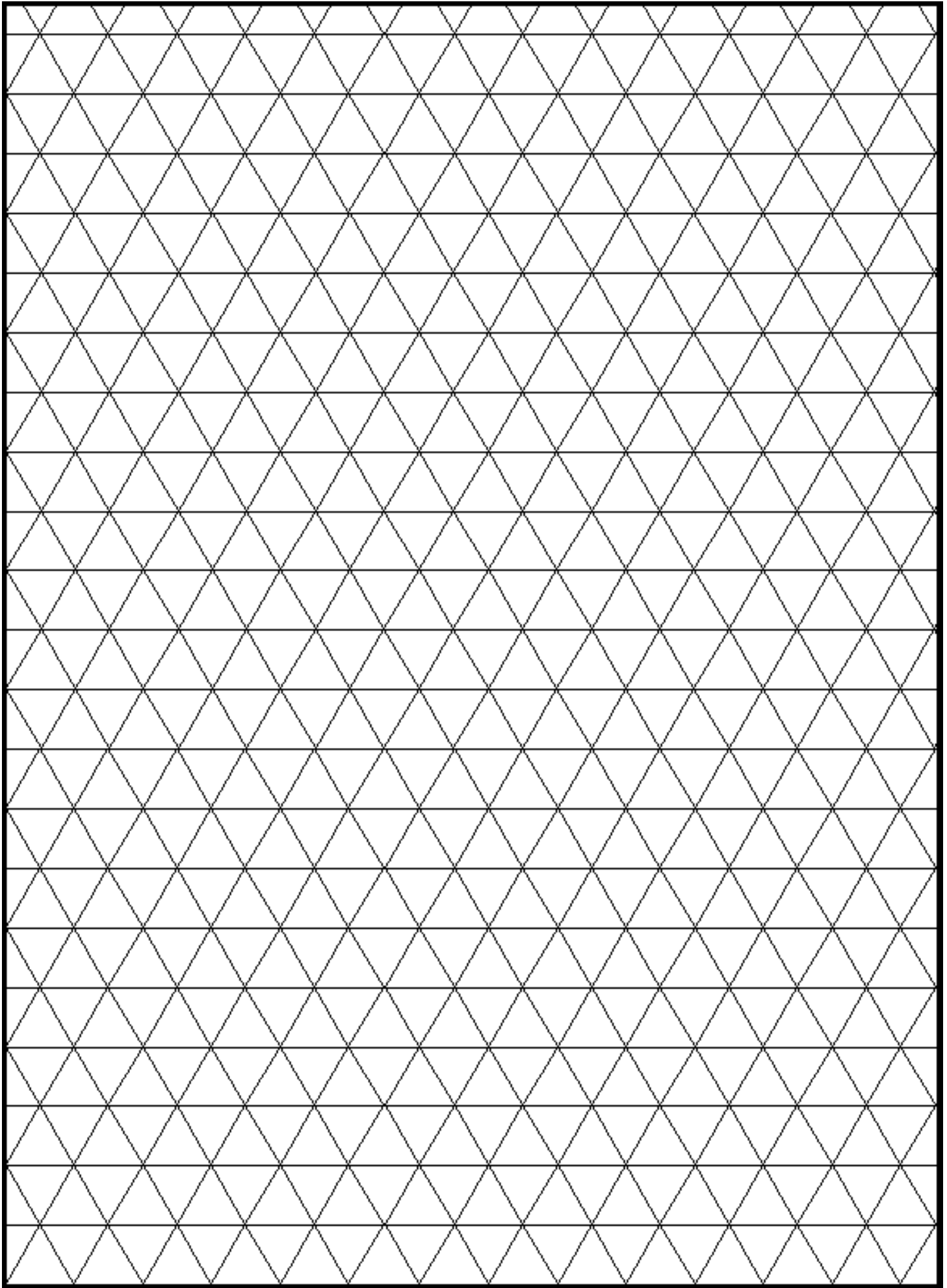
Take screen shots of these and save in Photo Library.

Import these photos into iMovie.

Write a script explaining what fractals are and what you did? Include the magnifications and your observations about the fractals.







Pythagoras and his Amazing Triangle

Everything = Number = Pythagoras

Pythagoras lived about 500 BC, that's about 2,500 years go.

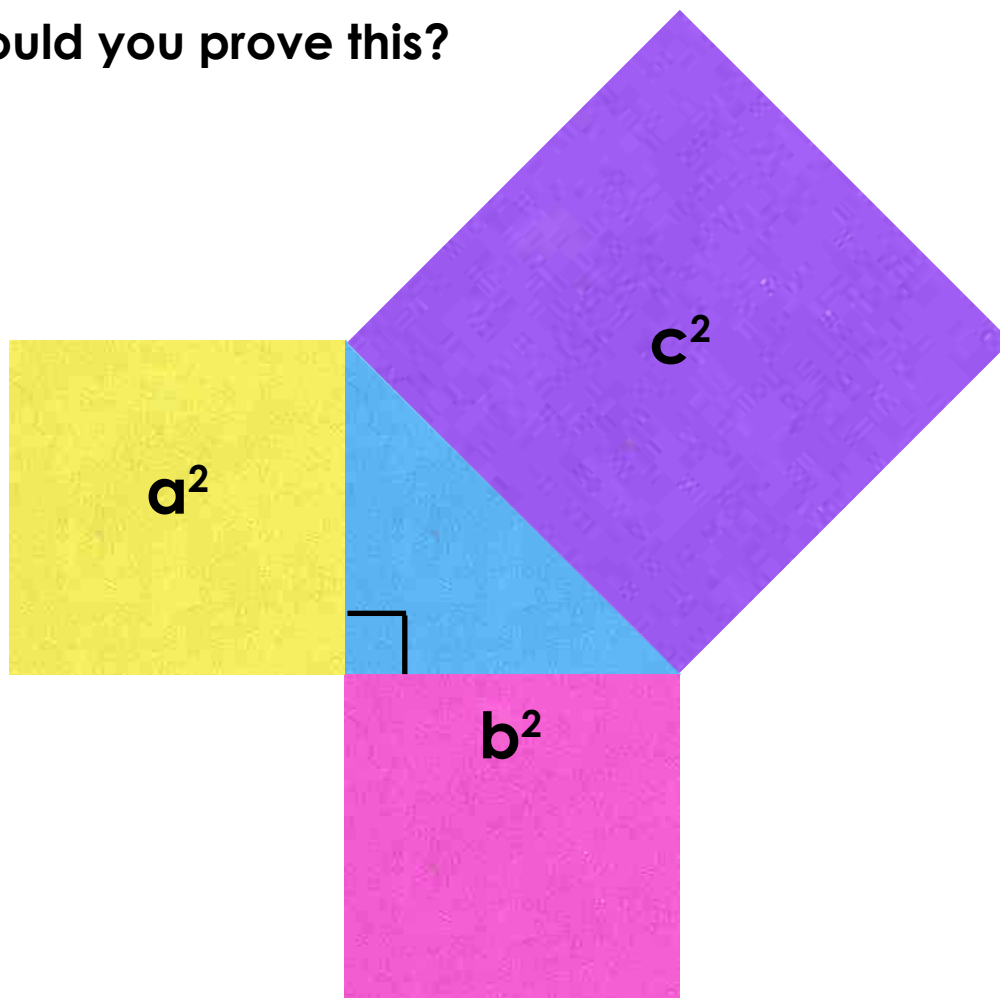
He was Greek philosopher and founder of a secret religion. Some of their secret rules were Do not to eat beans, Do not touch a white rooster. Do not stir the fire with iron.. Do not walk on highways. Do no let swallows nest in your roof.. Do not look in a mirror if it is beside a light. Make your bed when you get up in the morning. Would you like to be part of his secret maths society? He's most important discovery is the equation $a^2+b^2=c^2$

In addition, Pythagoras believed that "All is number."and the Pythagoreans gave numerical values to many objects and ideas. These numerical values, in turn, were endowed with mystical and spiritual qualities.

Language

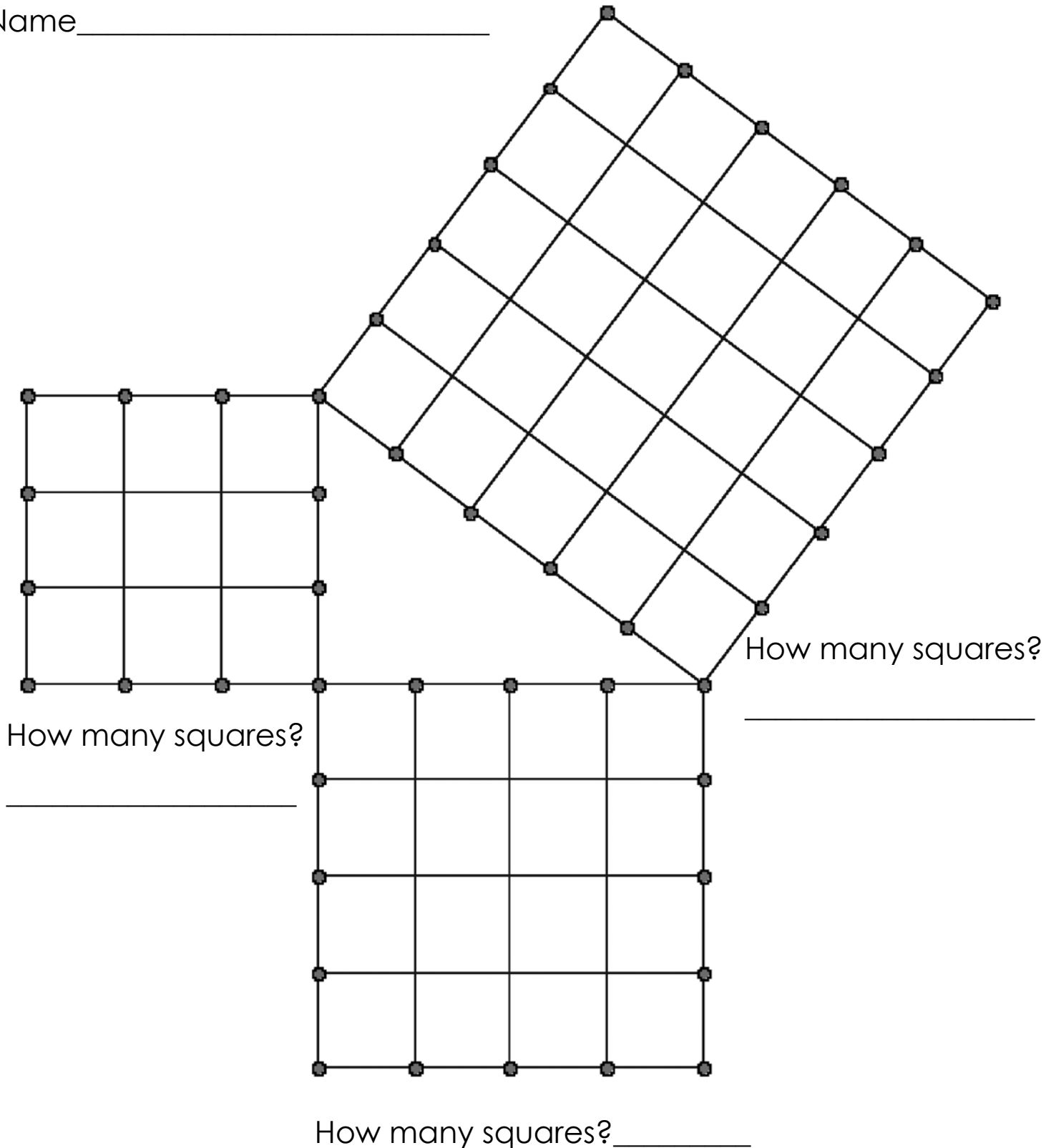
sides, area, vertices, hypotenuse, right angle, triangle, isosceles, square, equal, acute angle, 90 degrees,

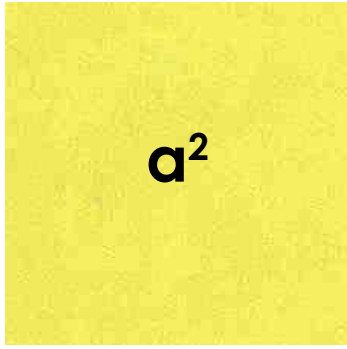
How could you prove this?



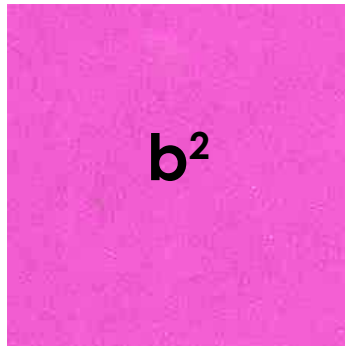
Pythagoras Triangle

Name _____

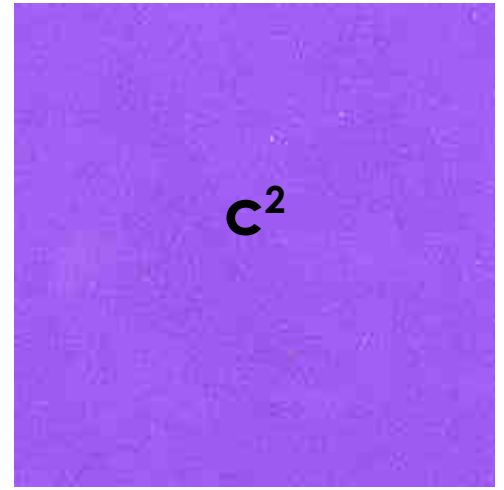




+



=

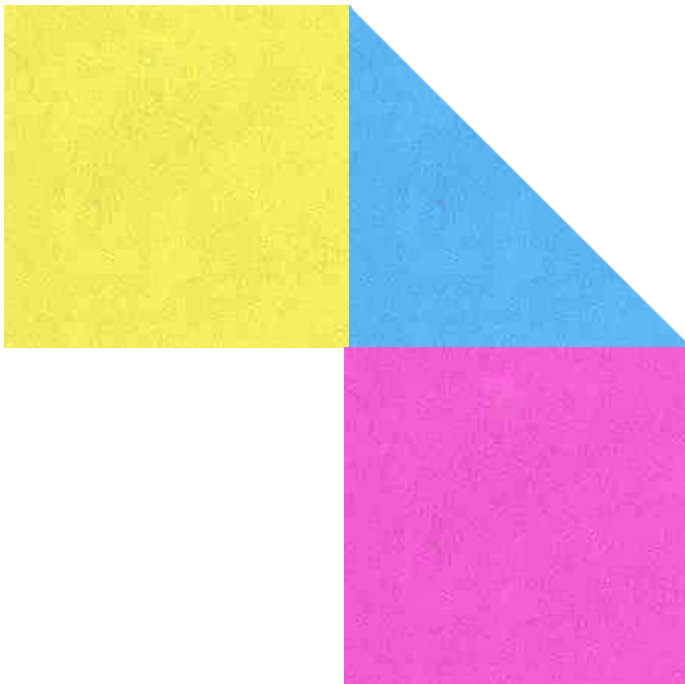


Provide Stage 1 with 5 small construction squares - mine are about 127mmx127mm - get them to cut one of the squares in half along the diagonal. This will give you the amazing Pythagorean Triangle you need to start with.

Then glue the squares onto the shorter 2 sides.

NOW can you work out how to make the square on the third side - the hypotenuse using 2 of the squares.

Can you experiment with this and figure out a solution?



1. Possible solution is below, can you find others?

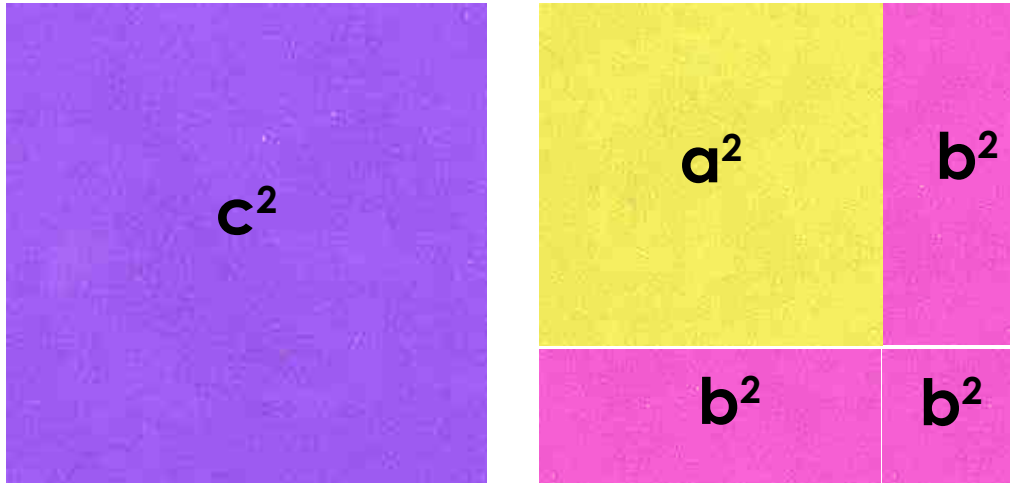
According to some there are over 600 proofs of Pythagoras Theorem - Cut the Knot has quite a few <http://www.cut-the-knot.org/pythagoras/>

A visual representation and also additional info on Pythagoras is at:-

<http://www.mathsisfun.com/pythagoras.html>

and also an amazing animated gif at

http://en.m.wikipedia.org/wiki/Pythagorean_triple#/image/File:Pythagorean_theorem_-_Ani.gif



Use some of the templates on the following pages to work on other solutions.

Challenge if your up for it

Google other solutions and demonstrate them.

Great Book - What's Your Angle, Pythagoras? by Julie Ellis (Stage 1 + 2)

<http://www.amazon.com/Whats-Angle-Pythagoras-Julie-Ellis/dp/1570911509>

Video "What's Your Angle, Pythagoras?" by Julie Ellis

<https://www.youtube.com/watch?v=dKcKbX1diOI>

Video Horrible Histories - Stupid Deaths (Stage 2 -Stage 3)

<https://www.youtube.com/watch?v=iBqEpC-dHqk>

Pythagoras Theorem Calculator for tricky square roots (Stage 3) algebra.

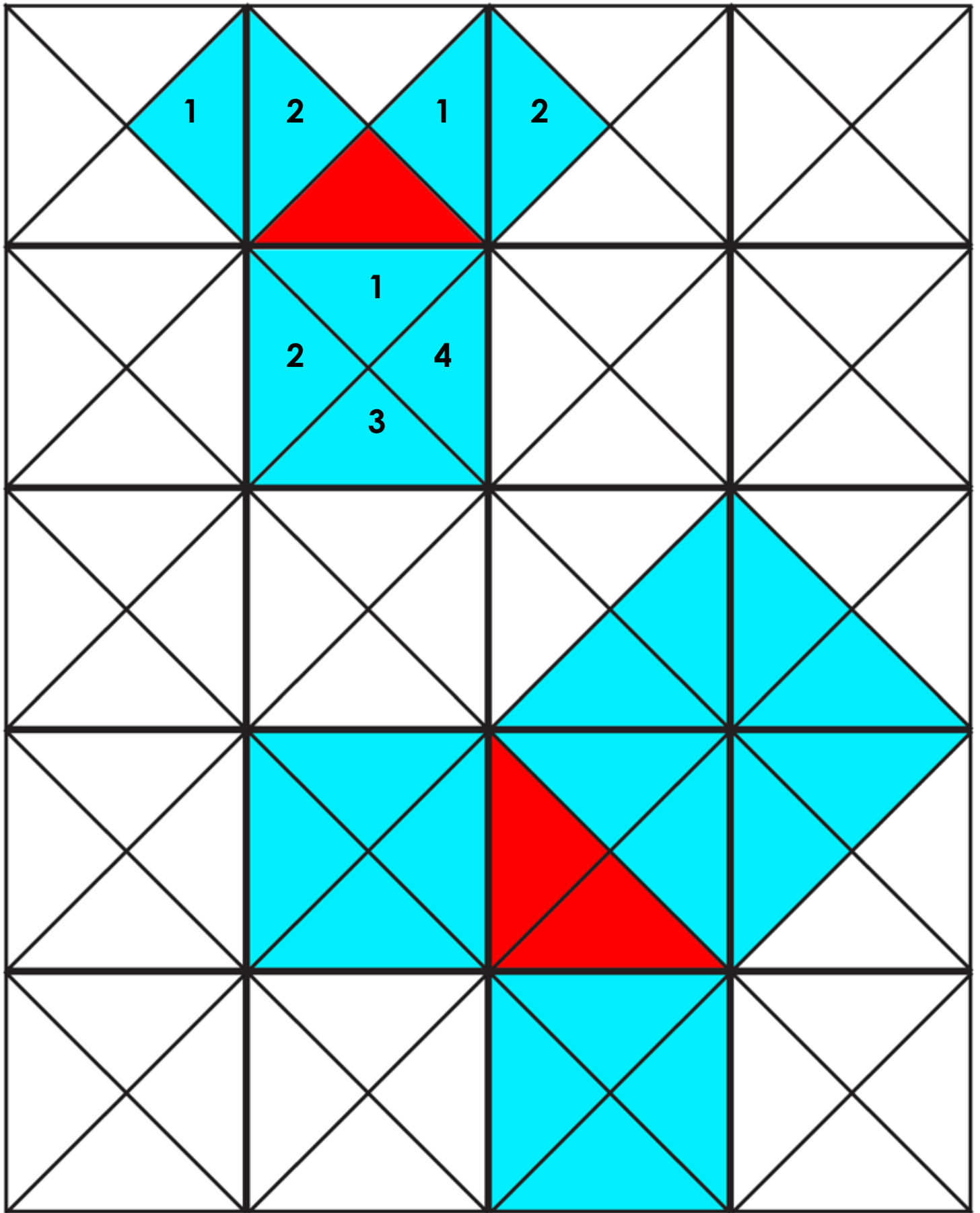
<http://www.miniwebtool.com/pythagorean-theorem-calculator/?n1=1&n2=&n3=2>

This is great for working on the pythagorean triples and also real life uses of th Pythagorean Theorem. For instance - work out the hypotenuse for the monitor screen you are viewing. check out the screen for TVs. Also work out the unkown side of th triangle when you use a ladder.

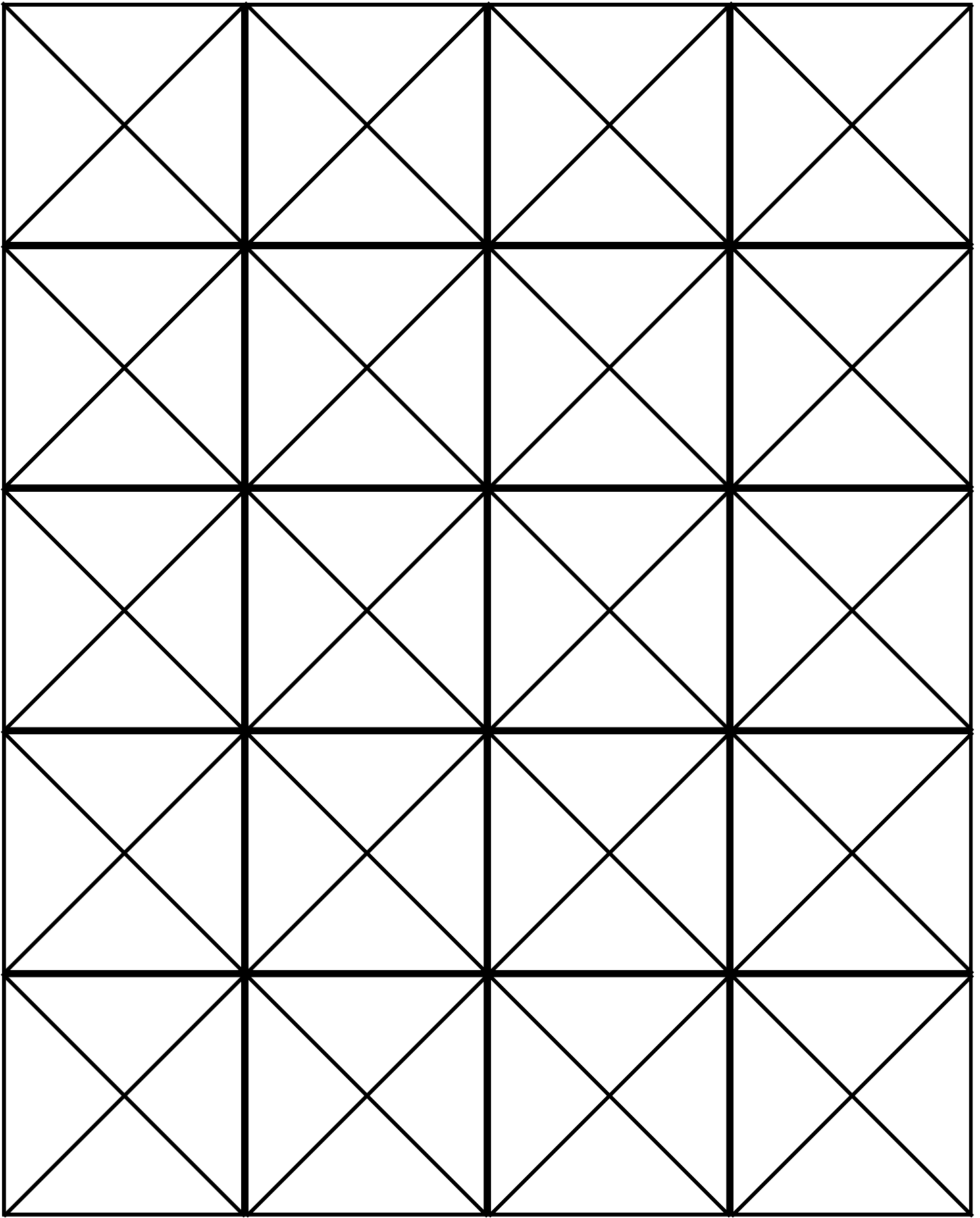
Other Things to Try

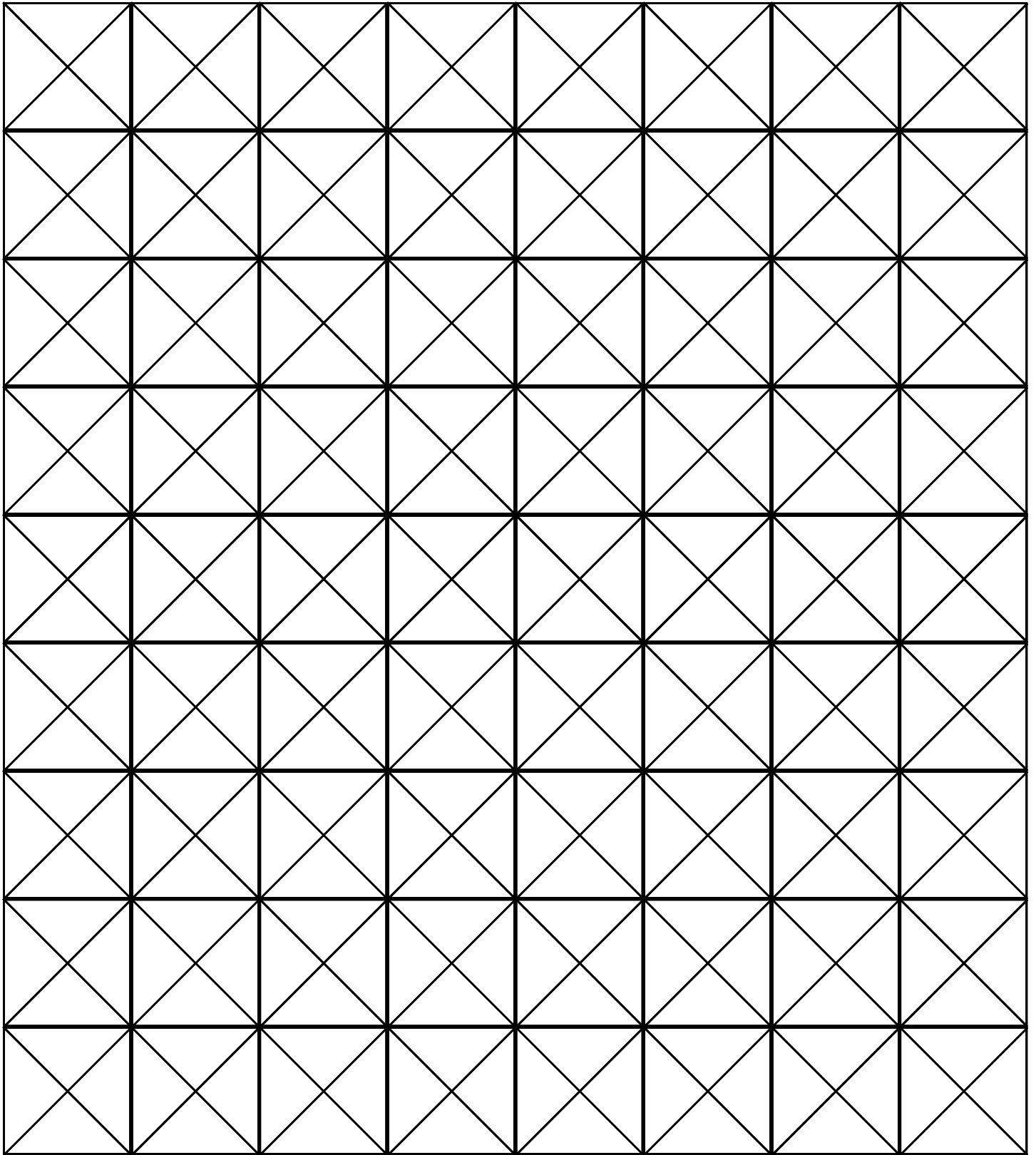
The most famous Pythagorean triple is 3,4,5. What is a Pythagorean triple. Can you work out some other?

Pythagoras also worked on Music and Tetracyts. Google these and figure out what he did.



Around 500 BC, the Greek mathematician Pythagoras discovered a rule which connects the lengths of the sides of all right angled triangles. It is thought that he discovered the rule while studying tessellations of tiles on bathroom floors. Such patterns, were common on the walls and floors of bathrooms in ancient Greece.





Make a Pythagorean Tree (Fractal)

The Pythagoras tree is a fractal constructed from squares.

Invented by the Dutch mathematician Albert E. Bosman in 1942, it is named after the ancient Greek mathematician Pythagoras because each triple of touching squares encloses a right angle triangle, which depicts the Pythagorean theorem. Check out the Pythagoras Tree Interactive at



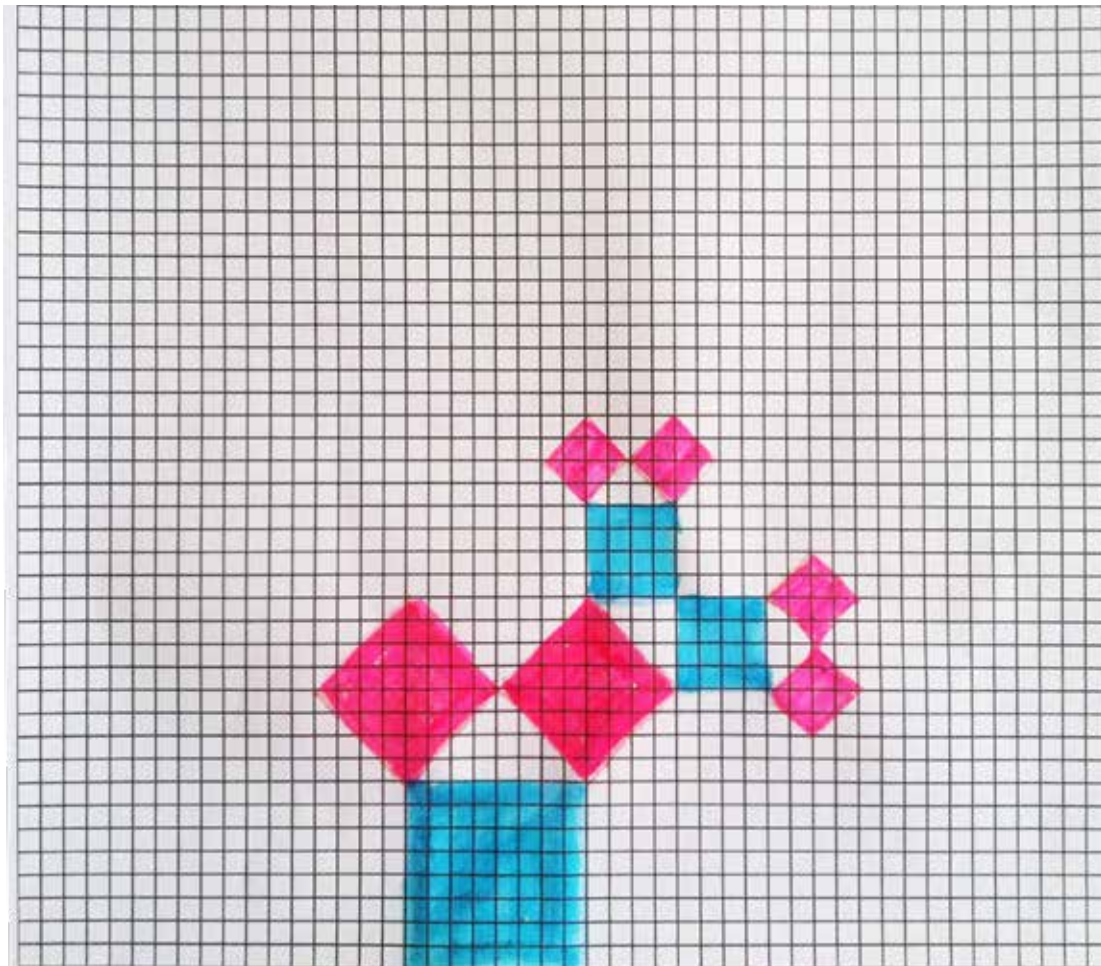
Also check out the FREE Pythagoras Tree app

<https://apps.apple.com/us/app/pythagorastree/id421785224>

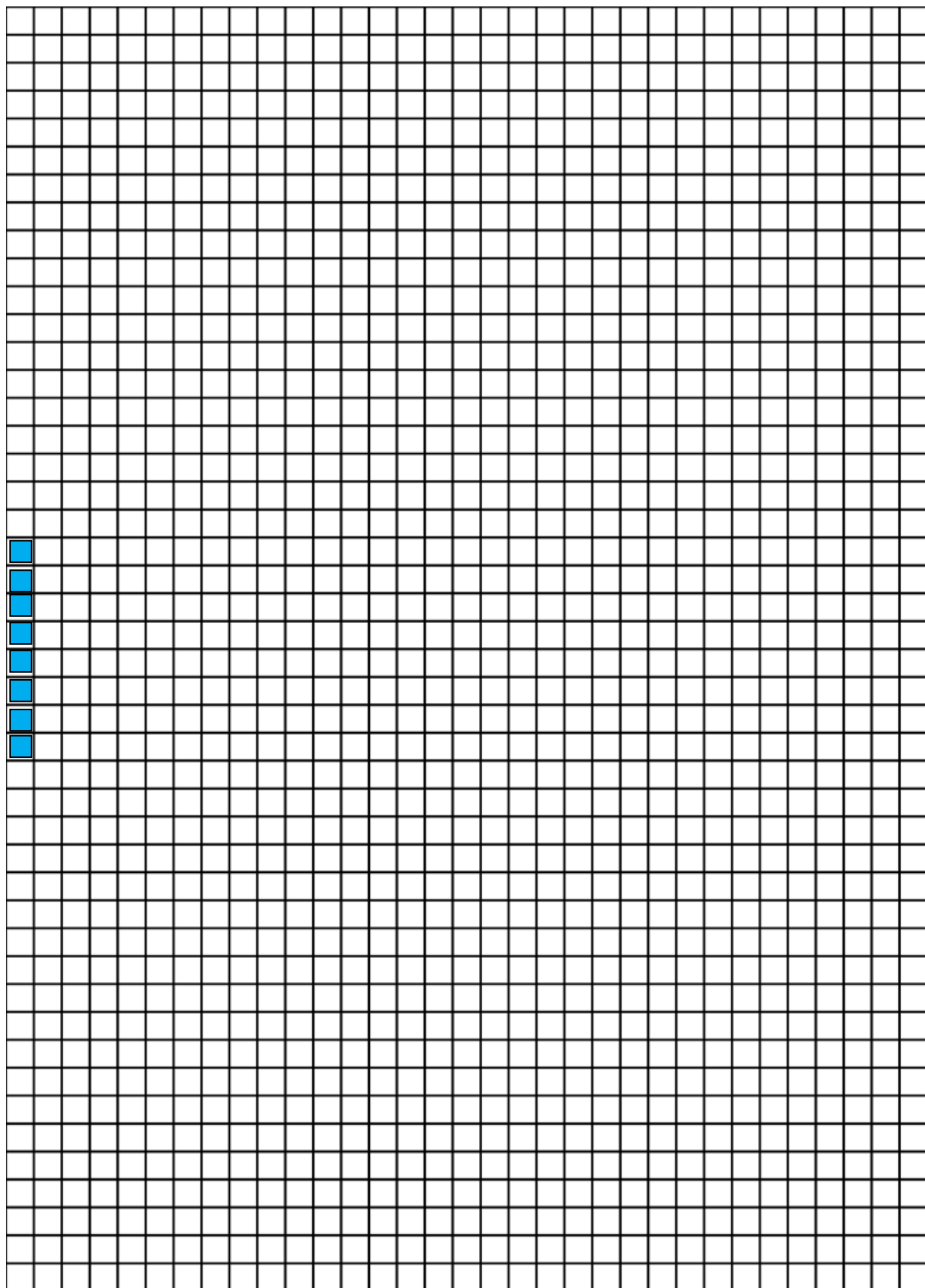
Take screen shots of your favourites and explain them in EXPLAIN EVERYTHING

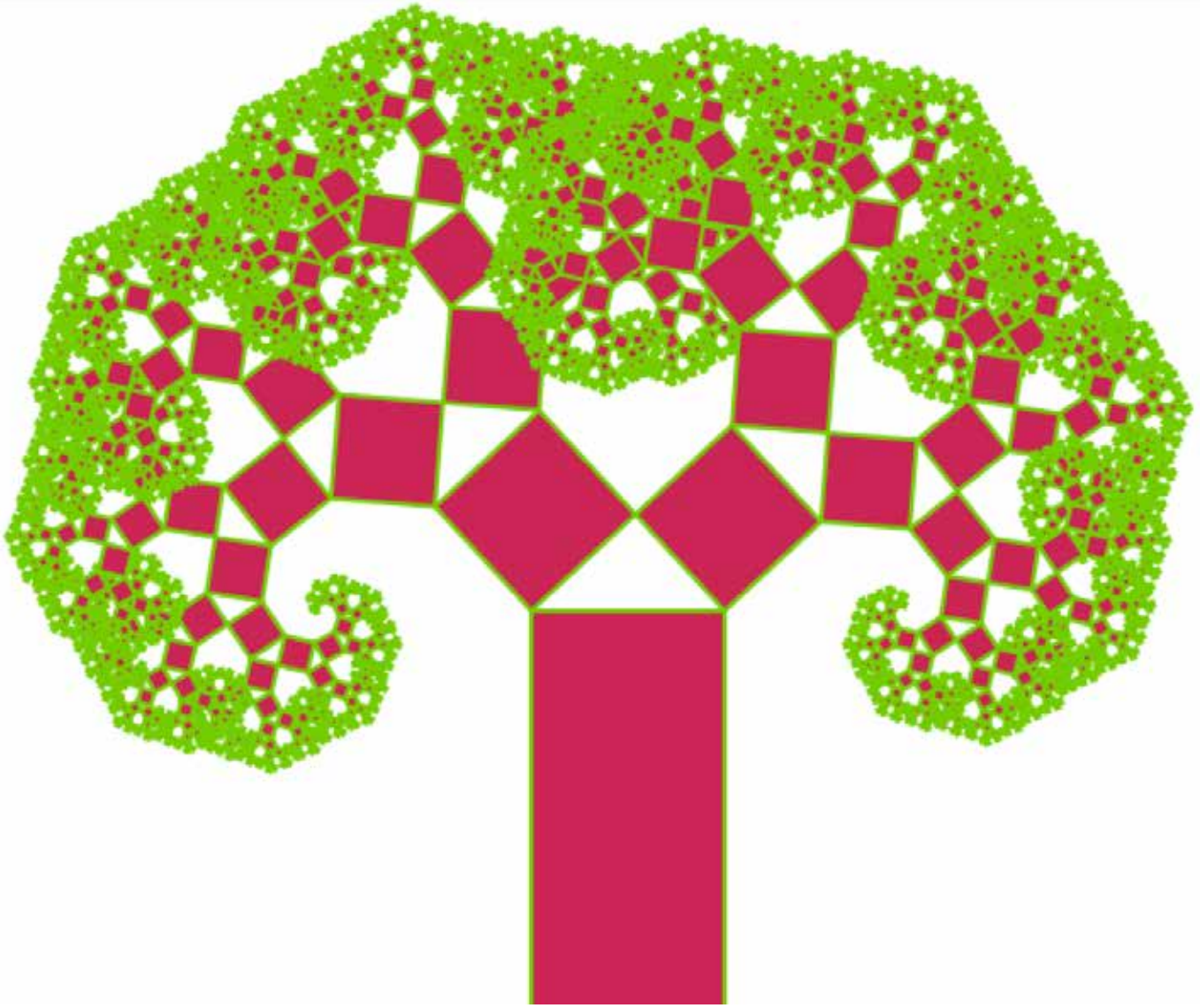
And YOU can make one. It works like this:-

Here's one started by Tom, you can use any grid paper you like - there is a sheet to follow. Tom started with an 8x8 square in the middle of the grid paper at the bottom, this works very nicely. Google some Pythagoras trees and check out the amazing colours they use. Choose your own colours and make a splendid art piece.



What other ways could you draw a Pythagoras Tree, invent one and see how small you can go. The above triangle is isosceles, the angles are 90° , 45° and 45° . Can you work





Pythagorean Tree

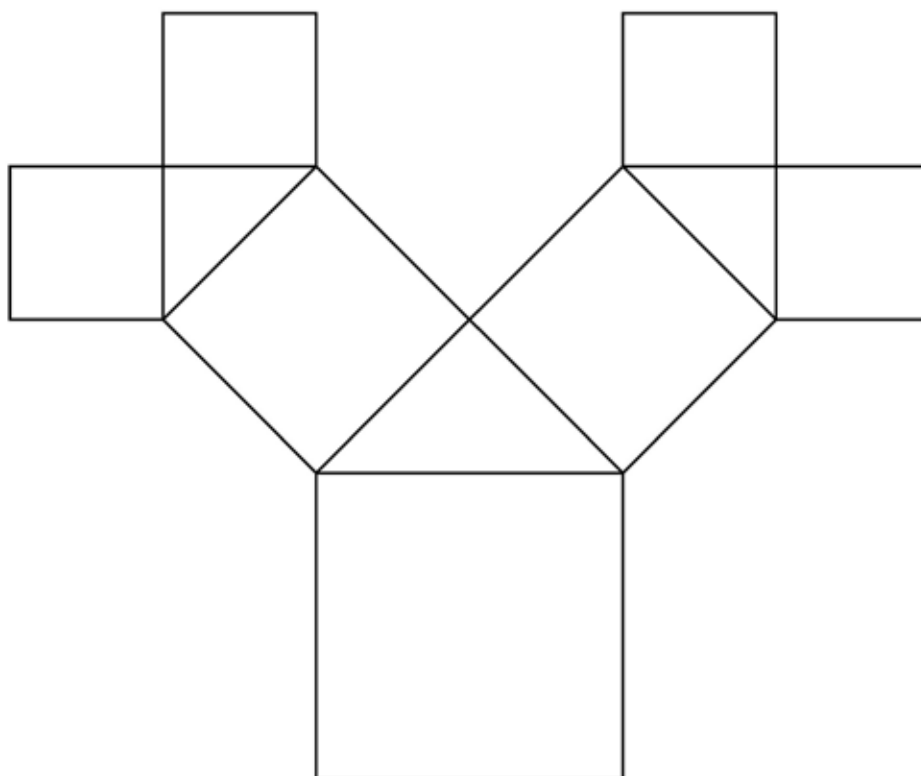
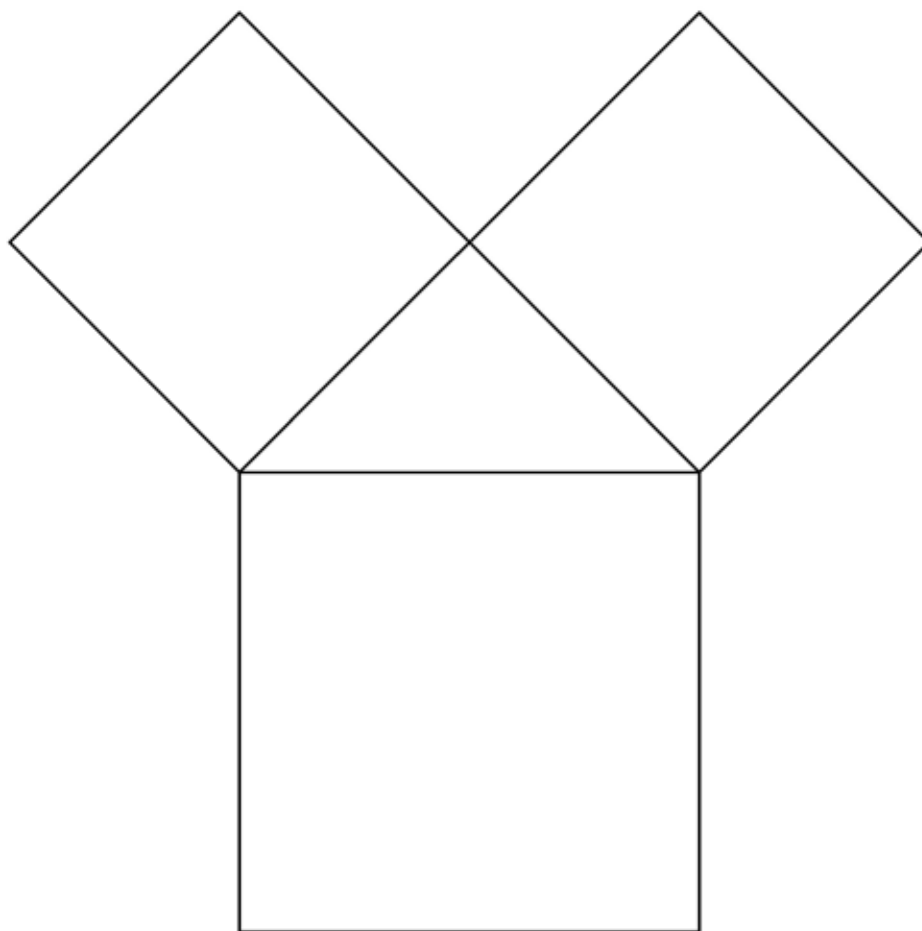
[https://upload.wikimedia.org/wikipedia/commons/1/18/Pythagoras tree 1 1 12 Jet.svg](https://upload.wikimedia.org/wikipedia/commons/1/18/Pythagoras_tree_1_1_12_Jet.svg)

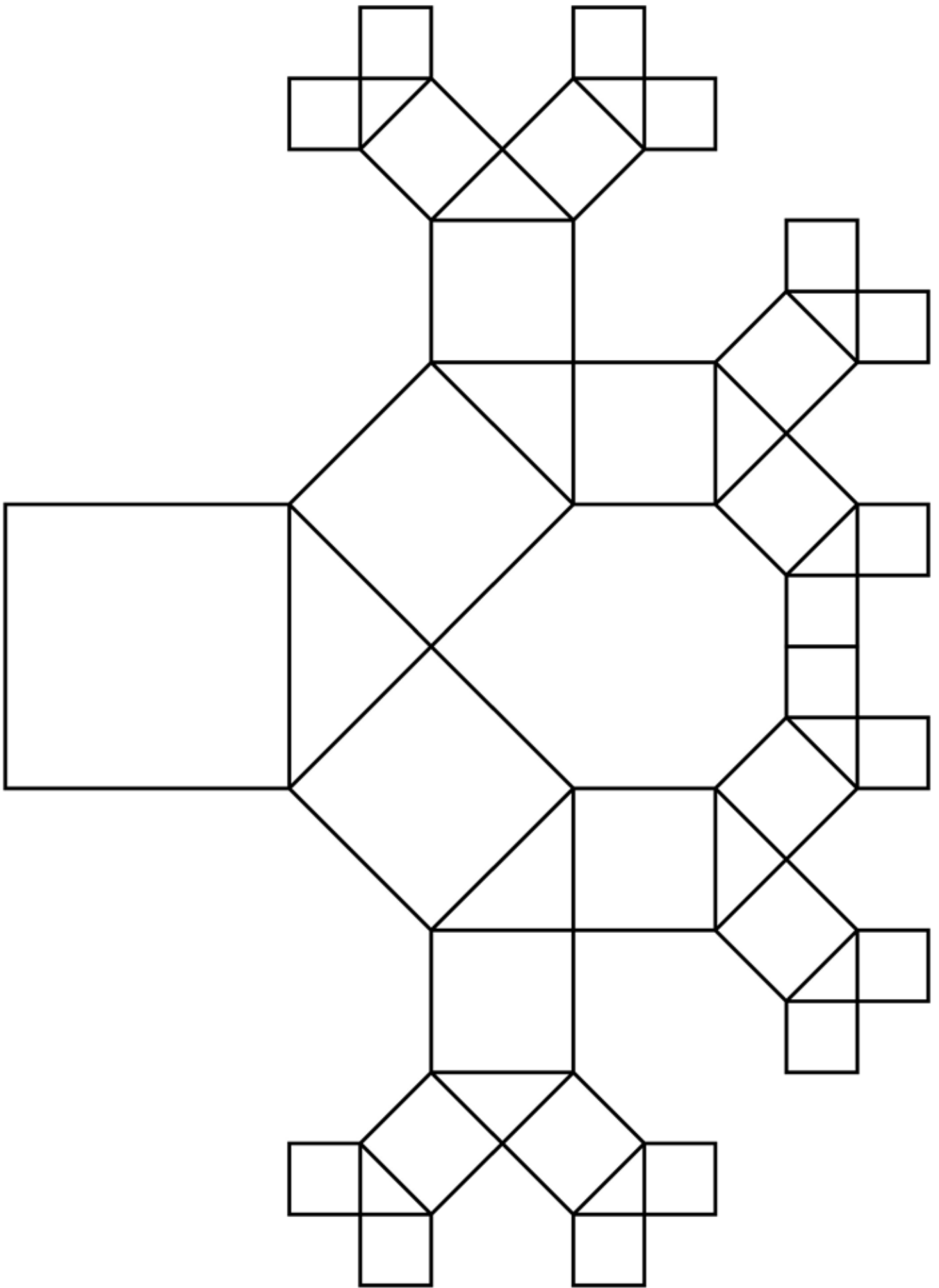
Pythagorean Tree Asymmetrical Wiki

[https://upload.wikimedia.org/wikipedia/commons/6/67/Pythagoras tree 1 2 12 jet.svg](https://upload.wikimedia.org/wikipedia/commons/6/67/Pythagoras_tree_1_2_12_jet.svg)

Pythagoras Tree on Scratch

<https://scratch.mit.edu/projects/10095046/>





4 Colour Theorem

How many colours do you need to colour a map so that touching regions are different colours? Why do you think this is important? Couldn't you just colour a map in all one colour?

Rules

Sections that contain a common edge or boundary must not be the same colour. However that two regions can be the same color if they meet only at a single point or vertex.

WATCH Simple Explanation of the 4 Colour theorem

<https://www.youtube.com/watch?v=UZXZhH012WI>

Language

boundary, edge, side, perimeter, vertex, point, corner, regions, area, colour, theorem, colour, map, parallel lines, right angle, rectangle, square, quadrilateral, equal, vertex, primary colours (red, blue, yellow) neutral colours (white, black, grey) vertical, horizontal.

To create maps that are easy to read, mapmakers often color them according to a rule that touching regions must always be colored differently. To color a large, complicated map this way, you might think you'd need to use a lot of different colors. But in fact, it has been proven mathematically that you never need more than four colors, no matter what the map looks like.

Francis Guthrie made this conjecture in 1852, but it remained unproven until 1976, when Wolfgang Haken and Kenneth Appel showed that it was true! Or is it?

Also, quite interestingly, this proof required the assistance of a computer to check 1,936 different cases that every other case can be reduced to! To date no one knows a quick short proof of this theorem.

The drawings below aren't all maps, but the same principle applies to them. Can you find a way to color all the regions in each drawing, using no more than four different colors, so that regions of the same color never touch (except at corners)? What is the least number of colours needed to colour each image?

Here's a handy hint: Before coloring a pattern, plan how you will do it by penciling in the names of your chosen colors in each region.

<http://www.mathpages.com/home/kmath266/kmath266.htm>

Even today, despite enjoying widespread academic acceptance, there are still skeptics who doubt the four colour theorem. Do you?

Can you come up with a counter-example?

4 Colour Theorem APP



FourColour App

is a fabulous and compelling app. It allows students to tap and color recolor spaces to solve the puzzles. Some quick thinking is required. Students can also make their own puzzles. You can also take screen shots of th before and after results and the original puzzles.

<https://itunes.apple.com/au/app/fourcolor-puzzle-four-color/id692422310?mt=8>

Best of all it's FREE it does have ads now - a bit annoying.

4 Colour Theorem Interactives

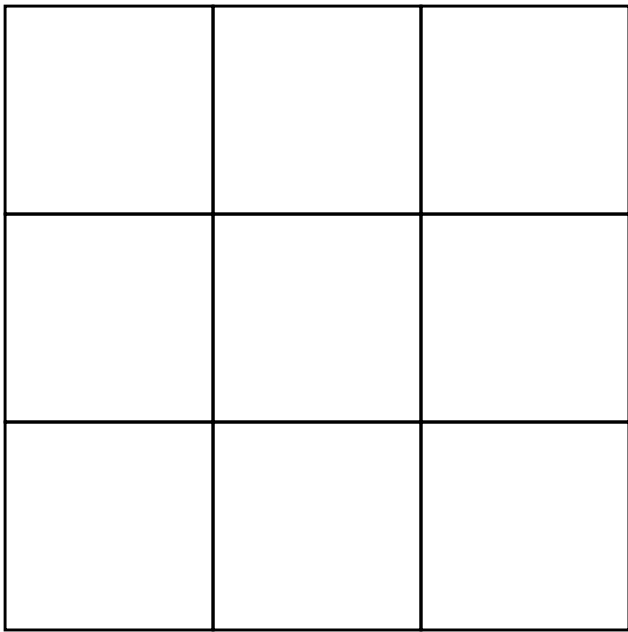
Animated 4 Colour interactive problems - great fun

<https://www.transum.org/Maths/Activity/Colouring/>

Paint the map with 4 colors so that the same colors do not touch on any one side.

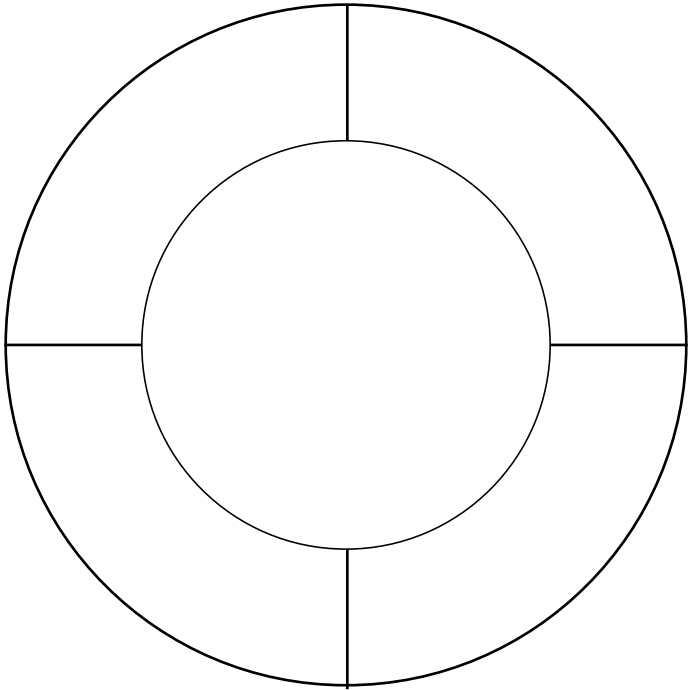
Mondrian 4 Colour Interactive

<https://demonstrations.wolfram.com/MondrianFourColoring/>



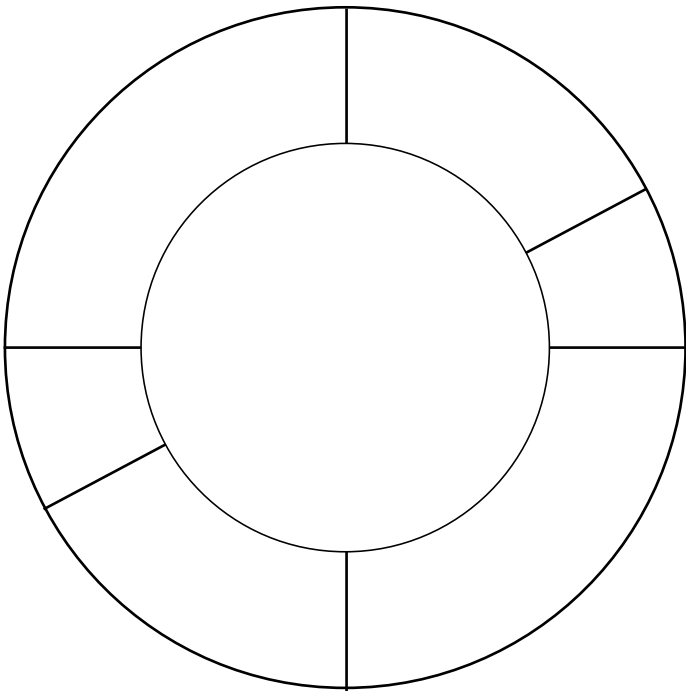
I predict I will use _____ colours.

I used _____ colours.



I predict I will use _____ colours.

I used _____ colours.



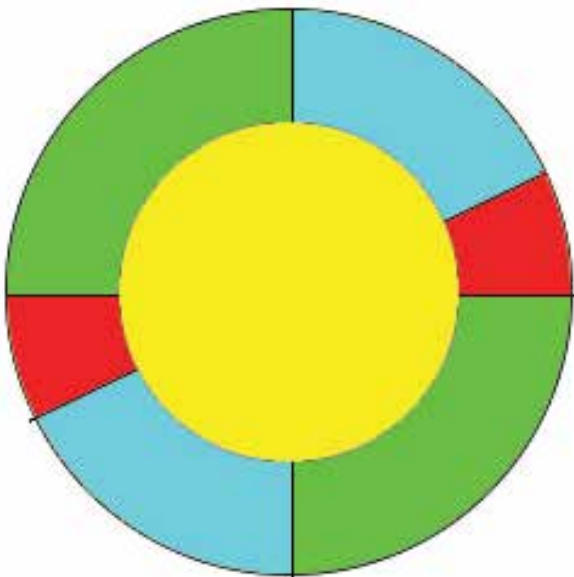
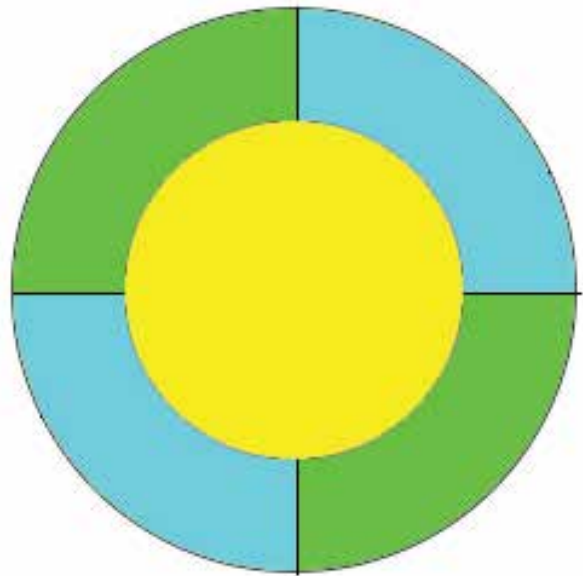
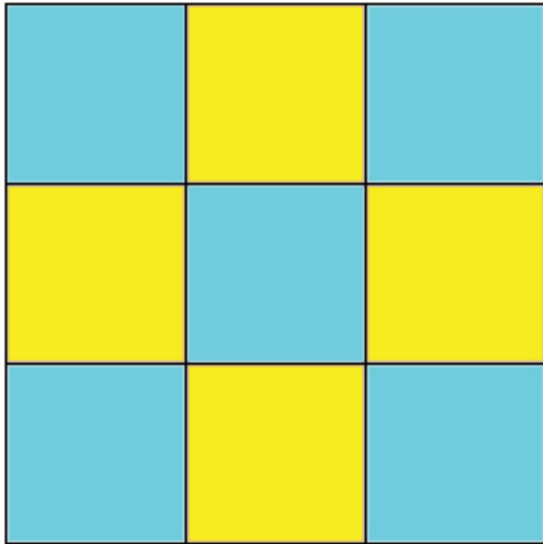
I predict I will use _____ colours.

I used _____ colours.



I predict I will use _____ colours.

I used _____ colours.



4 Colour Theorem & Art

Mondrian

Mondrian Four-Coloring Interactive

<http://demonstrations.wolfram.com/MondrianFourColoring/>

Create a Mondrian artwork using The 4 Colour Theorem

Look at works of Art by Mondrian on the WWW.

Composition A: Composition with Black, Red, Gray, Yellow, and Blue

1920 is an Excellent example - the artchive is a great site and you can scale the image to fit a white board.

http://artchive.com/artchive/M/mondrian/mondrian_composition_a.jpg.html

Great 1 sheet biog of Mondrian can be downloaded at: Free from

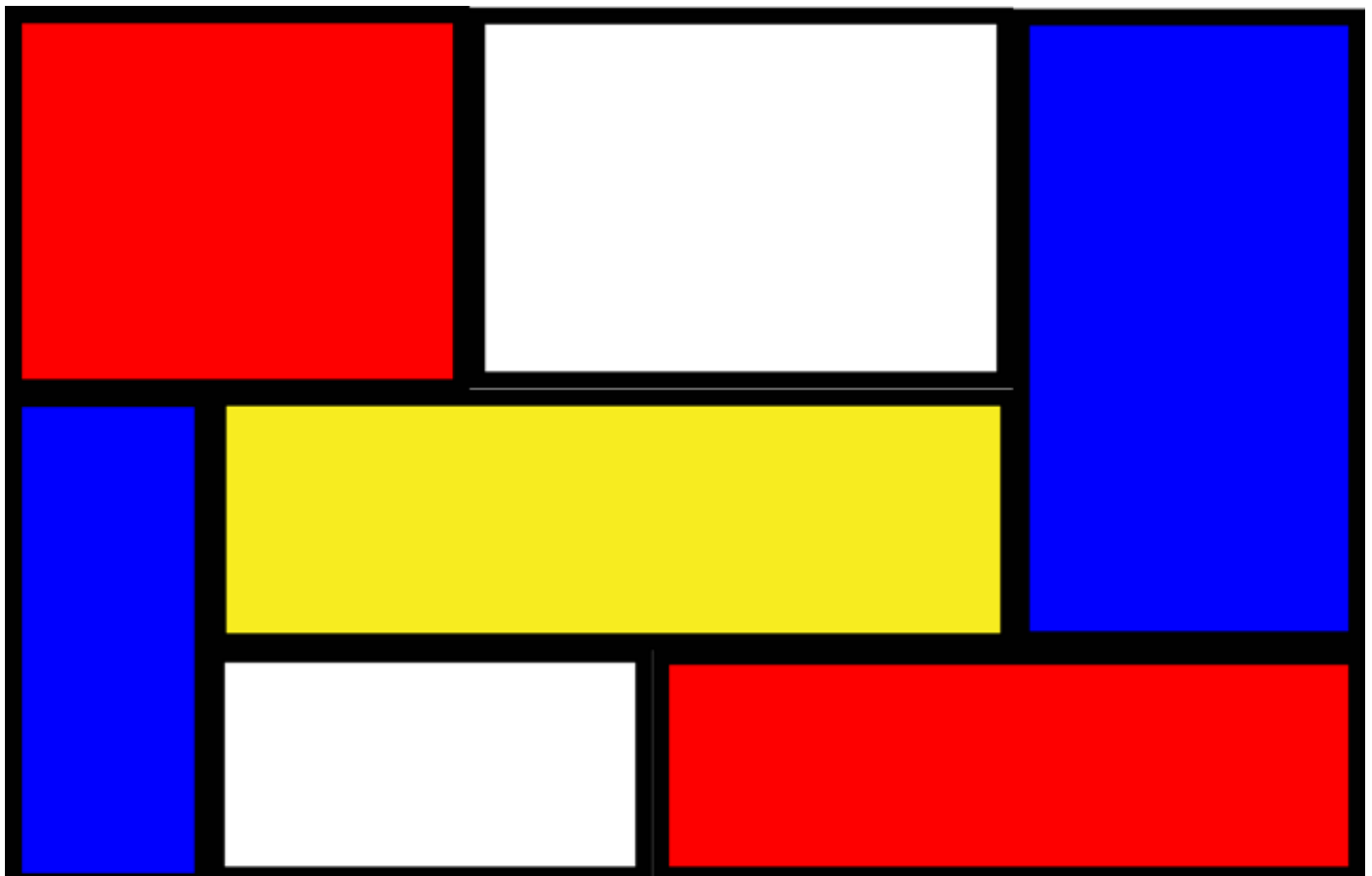
<http://makingartfun.com/html/f-maf-printit/piet-mondrian-print-it-biography.htm>

Mondrian used primary colours and neutrals for much of his artwork. Red, Blue, Yellow White and Black. He used only rectangles, squares and lines in many of his paintings. Make a MondrianEsq Artwork using only 4 of these options.

Technique 1 (you can also try this out with the shape tool in Microsoft word)

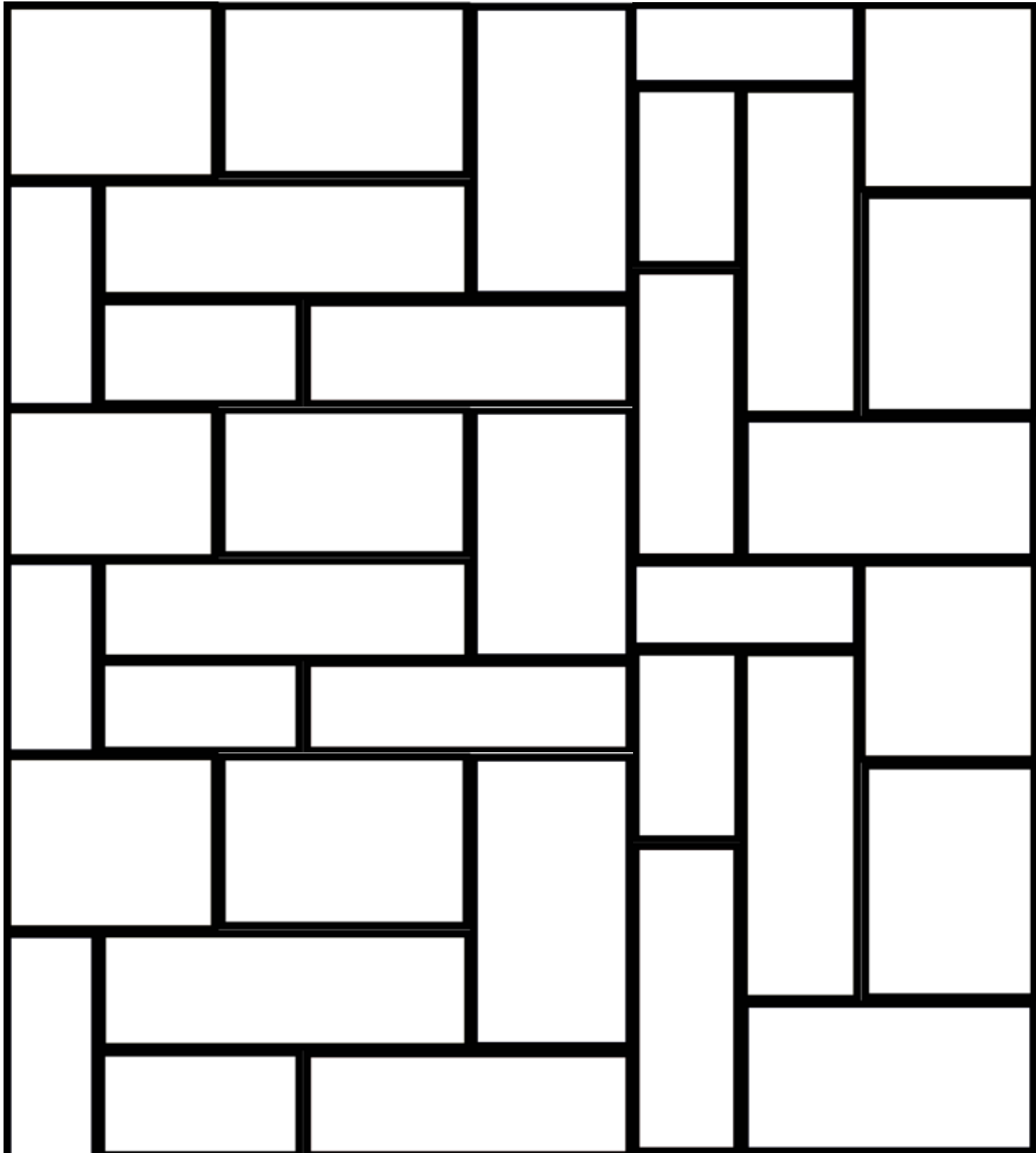
1. Cut out Coloured paper, edge the borders with black texta.

2. Glue them together onto black or white paper. Leaving some black or white squares on



4 Colour Theorem & Art

Mondrian BLM

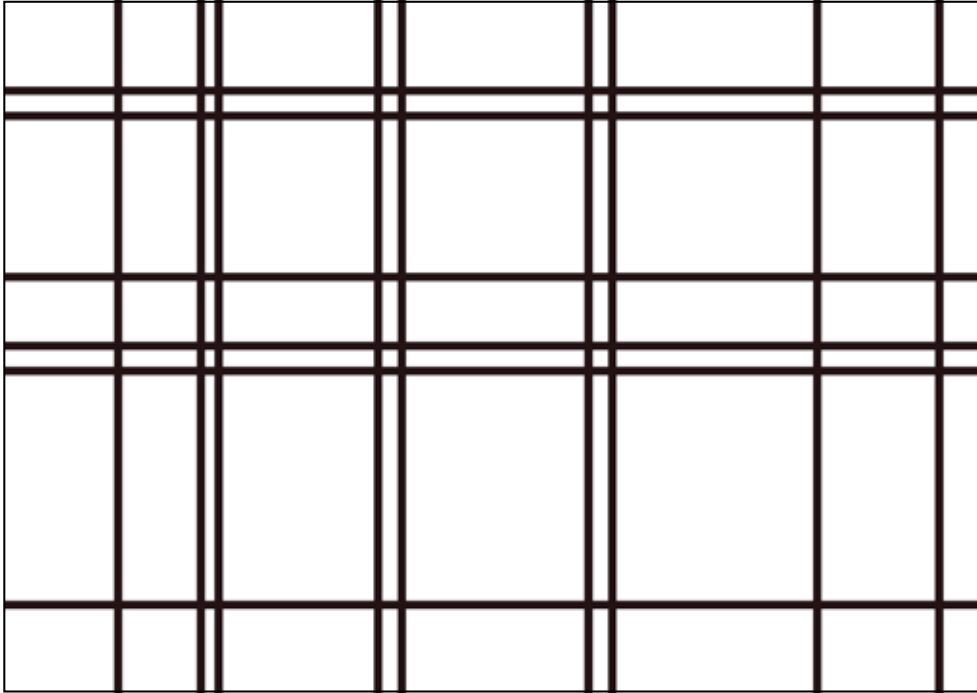


4 Colour Theorem & Art

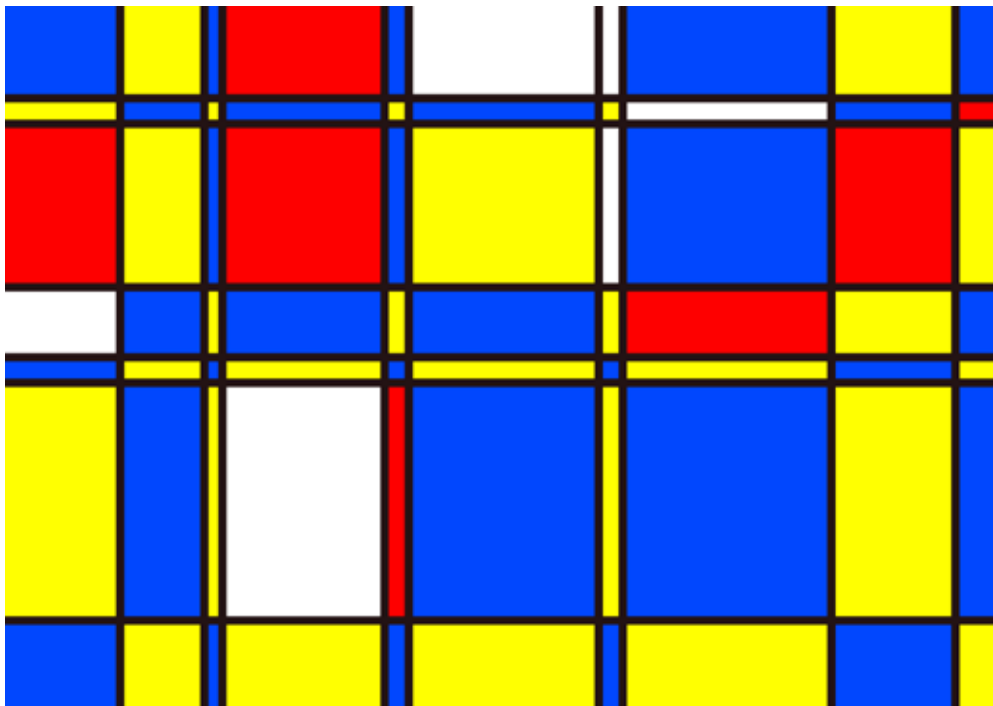
Mondrian

Technique 2

1. Draw lines on paper in black texta or dark pencil. Great practice for using a ruler. You should end up with something looking like this:-

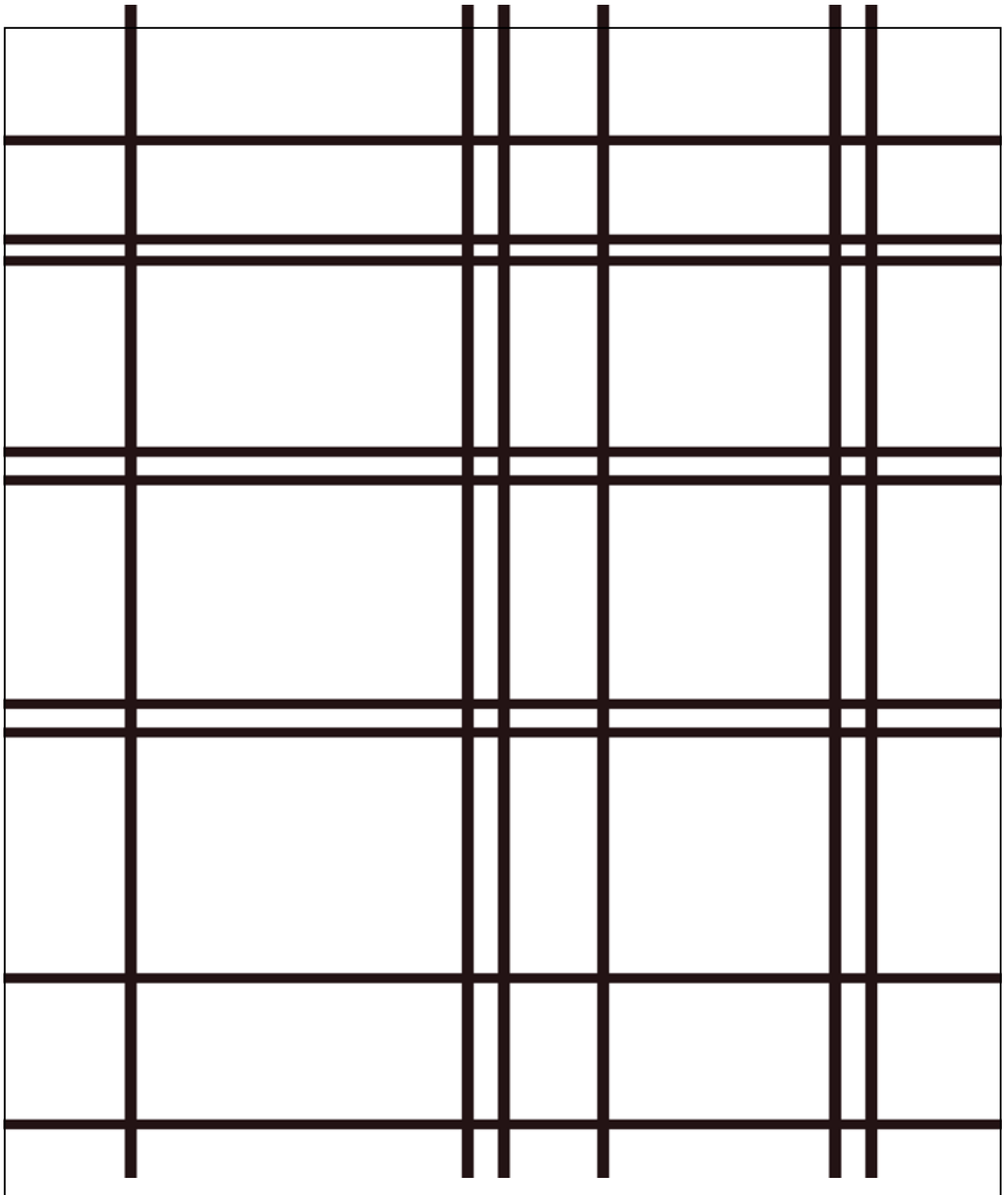


2. Then colour in the spaces using the 4 colour Theorem.



Technique 3

1. Print out this image and enlarge. Paint the primary colours using the 4 colour Theorem.
or trace the shapes onto coloured paper - bcut them out and glue them in place.



The Game of Col

If you play these games enough, you will be come so completely clever at them, that you can always have a strategy that will win (as long as you don't play them with someone who is quite as clever about them as you are.)

This game is taken from the book *On Numbers and Games* by John Conway (Academic Press, 1976) and is attributed to Colin Vout. Do you think the game is named after the inventor or for the fact that it has to do with coloring?

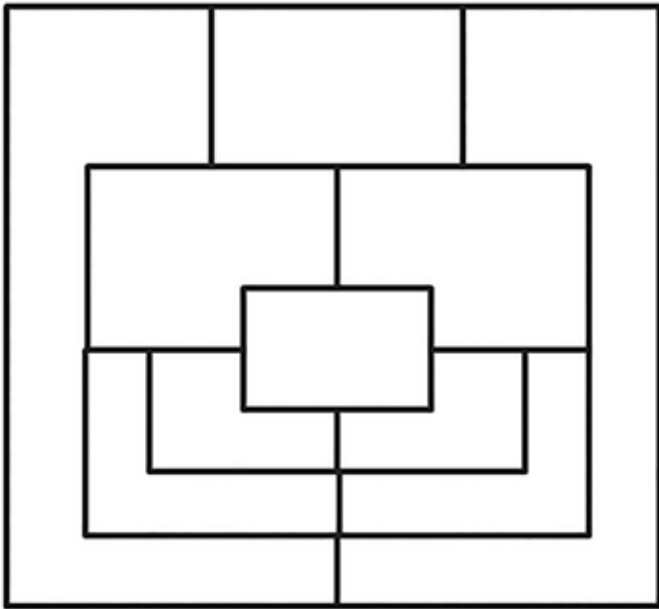
What you need

A map to color. (to follow)

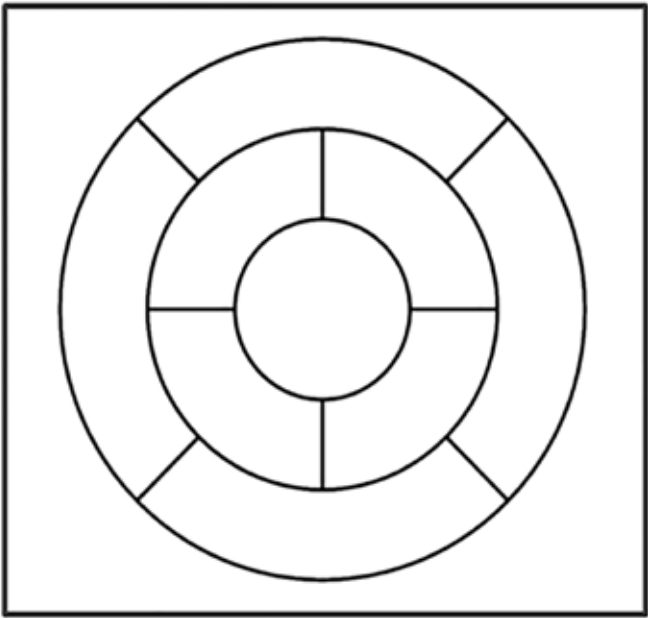
A different color of pencil for each of the 2 players.

How to Play

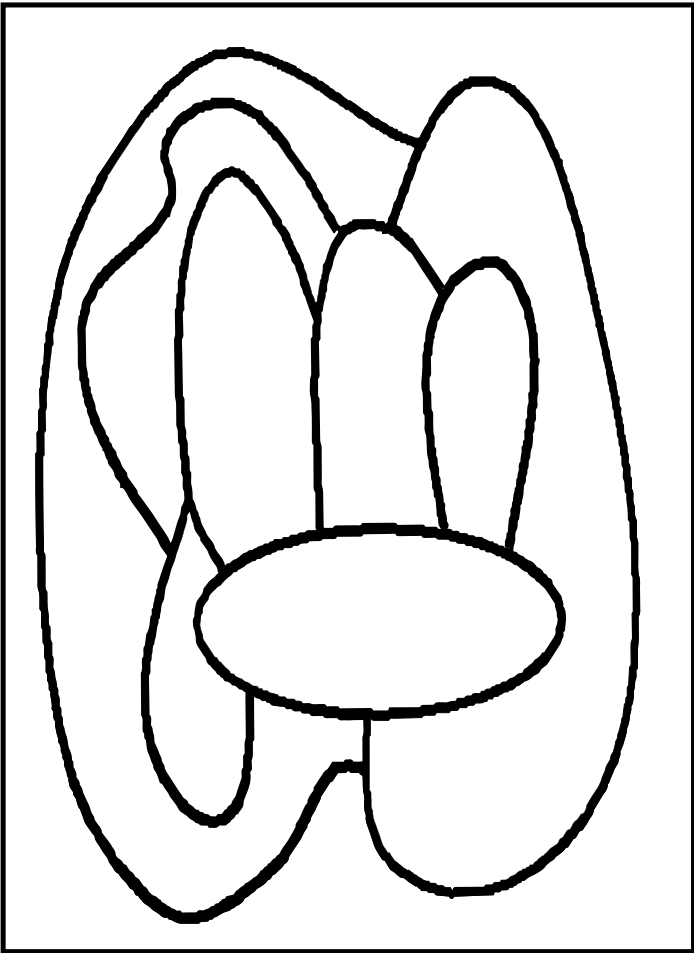
1. The first player colors any region of the map, and uses that color throughout the game.
2. The second player uses a different color and colors any region of the map. The second player also keeps the same color throughout the game.
3. The first player colors a region of the map, but cannot color a region that would cause 2 regions that share a boundary to have the same color.
4. The second player colors a region, but again , it must be a region that does not share a boundary with a region that is already colored in the color that the second player is using.
5. Notice, however, that two regions can be the same color if they meet only at a single point. A boundary has to be a line.
6. The game ends when one of the players is unable to find a region to color. That player loses.



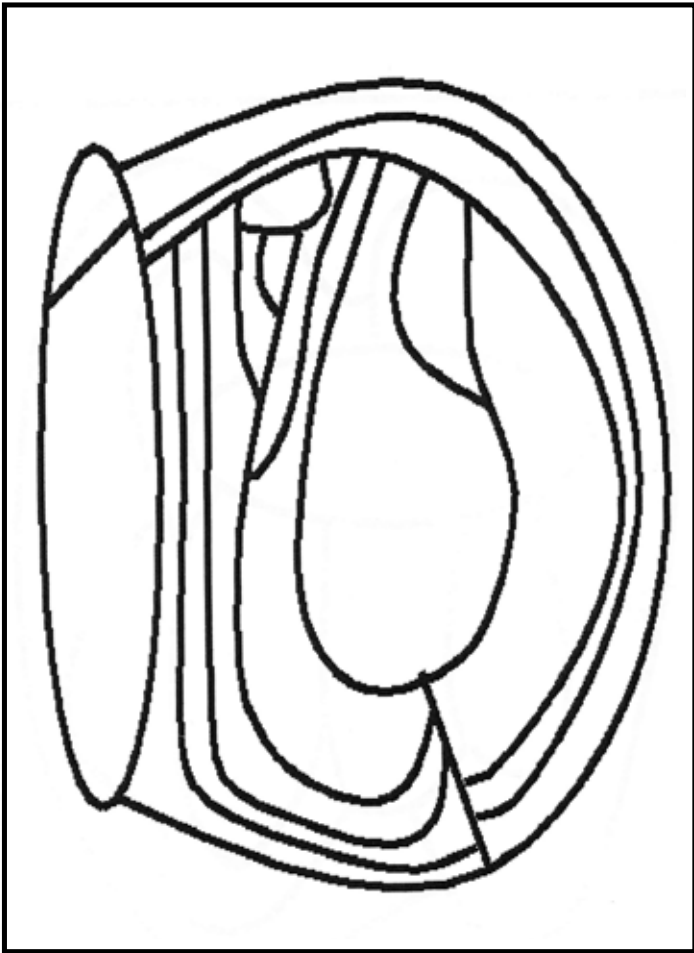
Winner_____



Winner_____



Winner_____



Winner_____

Can you draw your own maps on the back of this sheet and play? What would happen if you had 3 different players or 4 different players each with thier own colour?

Magical Fibonacci

Math + Nature = Fibonacci

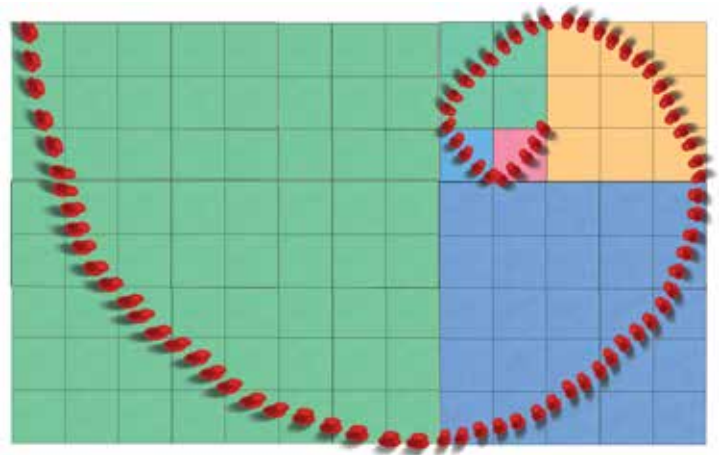
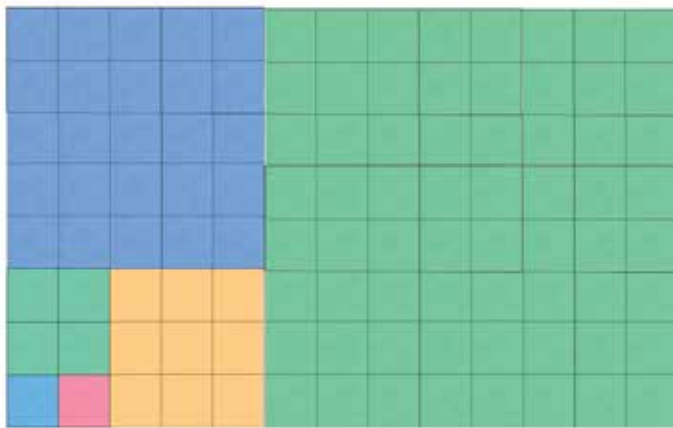
I love Fibonacci, how amazing, his numbers explain how nature works. What an amazing concept.

Kindergarten -Stage 1

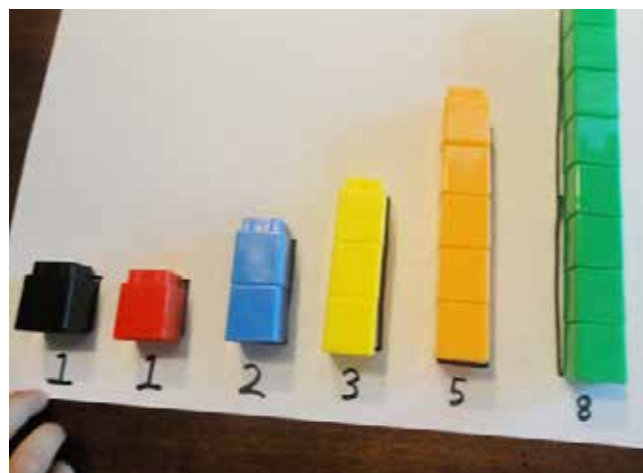
Fibonacci in Nature - Collecting Flowers, pine cones
Make Fibonacci Flowers



Make a Fibonacci spiral as a whole class using postit notes
Trace the spiral with thumb tacks



Represent Fibonacci
Numbers with
building blocks



Read

Blockhead: The Life of Fibonacci by Joseph D'Agnese

Buy the book or view it online

Blockhead The life of Fibonacci

https://www.youtube.com/watch?v=YFKciHHYH_I

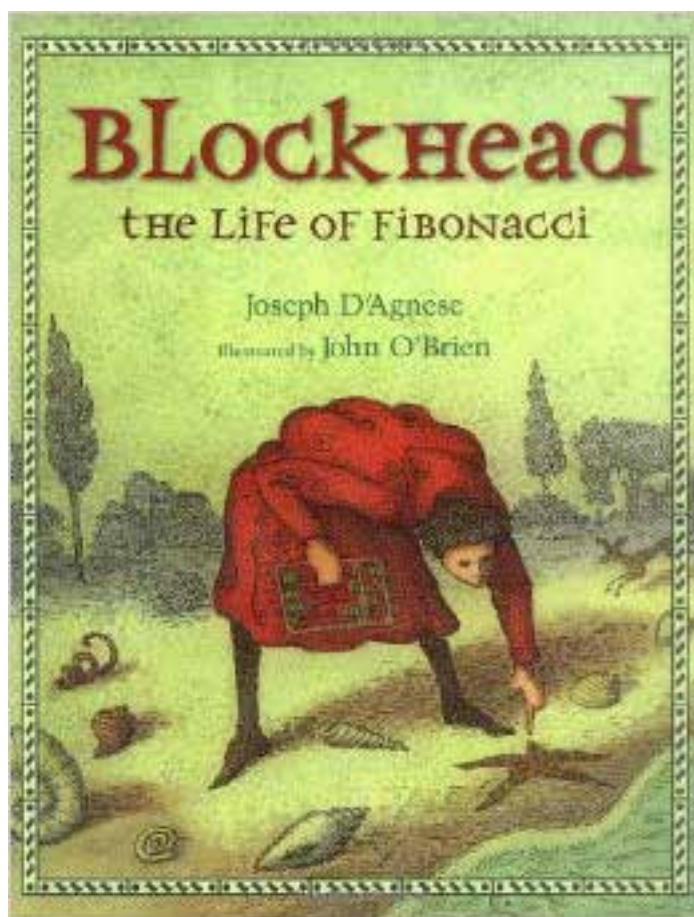
Book Trailer <https://www.youtube.com/watch?v=XltCNf5Bjew>

Synopsis

As a young boy in medieval Italy, Leonardo Fibonacci thought about numbers day and night. He was such a daydreamer that people called him a blockhead.

When Leonardo grew up and traveled the world, he was inspired by the numbers used in different countries. Then he realized that many things in nature, from the number of petals on a flower to the spiral of a nautilus shell, seem to follow a certain pattern.

The boy who was once teased for being a blockhead had discovered what came to be known as the Fibonacci Sequence!



Some Mathematical concepts covered in the book

- What is a number?
- Hindu-Arabic numerals versus Roman numerals
- The Value of Place Value
- Egyptian numerals
- Using an Abacus
- Travel distances (Pisa to Bugia and other sites in the Mediterranean)
- Number patterns
- Fibonacci numbers in Nature
- Fibonacci spirals

The author's Website - Why not send him an email?

<http://www.josephdagnese.com>

Arthur Benjamin: The magic of Fibonacci numbers (6.4mins) Some fabulous pattern finding. Stage 3 - Learning how to think. Figuring out why.

http://www.ted.com/talks/arthur_benjamin_the_magic_of_fibonacci_numbers

Fibonacci sequence

How To Calculate the Fibonacci Sequence

<https://www.wikihow.com/Calculate-the-Fibonacci-Sequence>

Fabulous explanation of Fibonacci Sequence

It includes the Golden Ratio, the numbers. The Spiral and the Algebra.

<http://www.mathsisfun.com/numbers/fibonacci-sequence.html>

Fibonacci Rabbits

<http://demonstrations.wolfram.com/FibonacciRabbits/>

Collect image examples of Fibonacci. Use Sketchup to Draw them

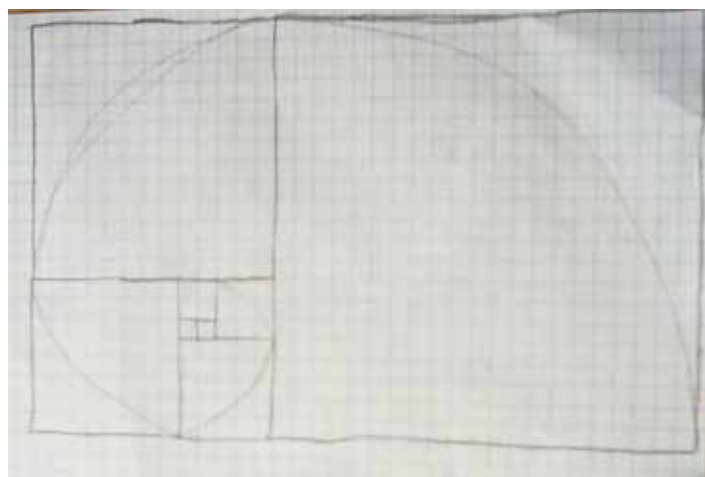
Draw a Fibonacci Spiral <http://www.wikihow.com/Draw-the-Golden-Spiral>

Draw a Spiral On Grid Paper - Great For Primary

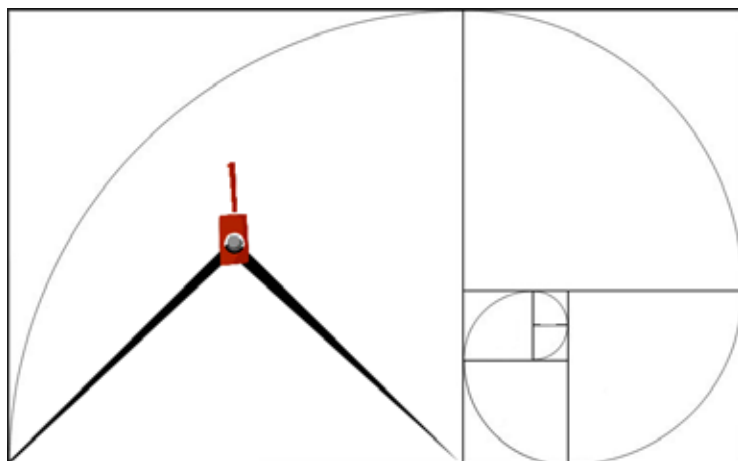
<http://www.mensaforkids.org/teach/lesson-plans/fabulous-fibonacci/>

Here's some work by my kid year 3s, they drew the squares and then sketched the circle
For older grades you can use a compass for more accuracy Check this out on Wiki How

<http://www.wikihow.com/Draw-the-Golden-Spiral>



Fibonacci Spiral drawn freehand



Fibonacci Spiral drawn with a compass

What is the Golden Ratio

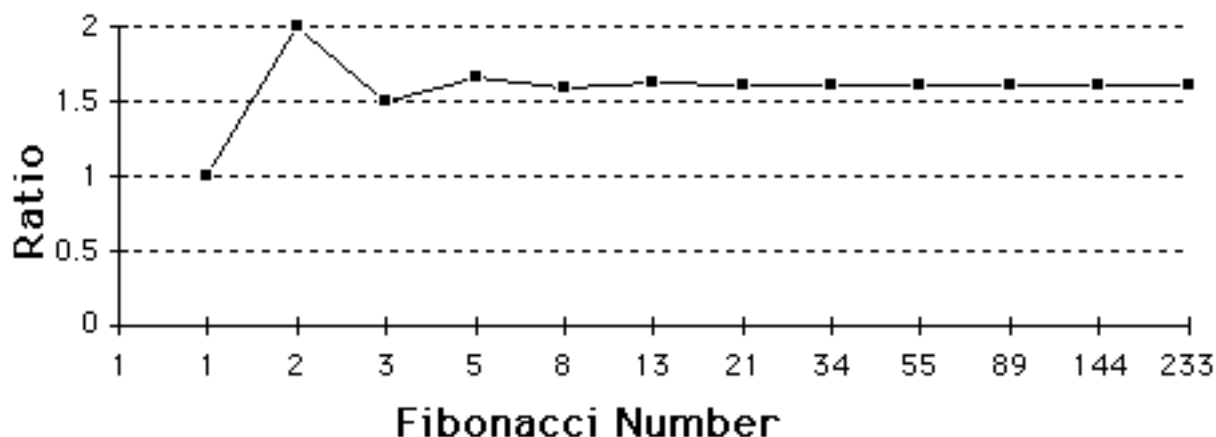
Fibonacci Numbers and the Golden Ratio

<http://demonstrations.wolfram.com/FibonacciNumbersAndTheGoldenRatio/>

Fibonacci Numbers and the Golden Ratio. Math concepts for kids.

<https://www.youtube.com/watch?v=CPTmRSYZupA>

RATIO OF SUCCESSIVE FIBONACCI TERMS



Some Fabulous Videos for V-Hart

Stage 2 and over Drawing the Fibonacci curve

Doodling in Math: Spirals, Fibonacci, and Being a Plant [1 of 3]

<https://www.youtube.com/watch?v=ahXIMUKSXX0>

Great Video showing how to tape a pineapple 3.40mins and see the spirals as well as how to draw a spirally pattern with shading 3.31mins

Doodling in Math Class: Spirals, Fibonacci, and Being a Plant [2 of 3] - Looking at the Golden Ratio - Stage 3 and over Measuring the angles between plant bits

https://www.youtube.com/watch?v=IOIP_Z-0Hs

Maths Forum - Fibonacci Numbers in Nature

The leaves here are numbered in turn each is exactly 0.618 of a clockwise turn (222.5°) from the previous one.

<https://insteading.com/blog/fibonacci-sequence-in-nature/>



Milky Way Galaxy (Pixabay)

Many of nature's patterns are related to the golden section and Fibonacci numbers.

For instance, the golden spiral is a type of spiral found in sunflowers, seashells, animal horns and tusks, beaks and claws, whirlpools, hurricanes, and **spiral galaxies**.

An equiangular spiral does not alter its shape as its size increases. Because of this remarkable property (known as self-similarity), it was known in earlier times as the '**miraculous spiral**'.

Pisano Period Advanced Stage 3 and up - Great activity for addition and division.

Make Music with Fibonacci - The Pisano period

<https://www.youtube.com/watch?v=Nu-lW-lfyec>

Divide Fibonacci by a number and write down the numbers left over - this will form a pattern.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711

Numbers are divided by 3 and the remainders are written below.....

0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2

What is the length of the period. For 5 it is 20. What patterns can you find?

Add the 2 remainders and get the next remainder. - amazing

Fibonacci Flower Petals



1 petal – white calla lily



2 petals - euphorbia



3 petals- iris

Images from [Pixabay.com](https://pixabay.com)

See if you can find examples of the rest.

5 petals - buttercup, wild rose, larkspur, columbine

8 petals - delphiniums

13 petals - ragwort, corn marigold, cineraria, black-eyed Susan

21 petals – Shasta daisy, aster, chicory

34 petals – field daisies, plantain, pyrethrum

55 or 89 petals - michelmas daisies, the asteraceae family

Fruits & Vegetables



A banana has 3 sections

An apple has 5 sections

Pineapple: The scales are patterned into spirals of hexagonal shapes of which three are distinct sets of spirals. One set of 5 spirals ascends at an angle to the right, a second set of 8 spirals rises steeply to the left and the third set of 13 spirals rises very steeply to the right.

Image from [Pixabay.com](https://pixabay.com)

You can go to town on Fibonacci artwork

Take look at:

Fibonacci night by Charles Stuart

<https://fineartamerica.com/featured/fibonacci-night-charles-stuart.html>

The double Spiral Devain Art Image

<https://s-media-cache-ak0.pinimg.com/736x/8a/e2/1f/8ae21f8a8c62016197222eb-fe41891a2.jpg>

Or the one below

